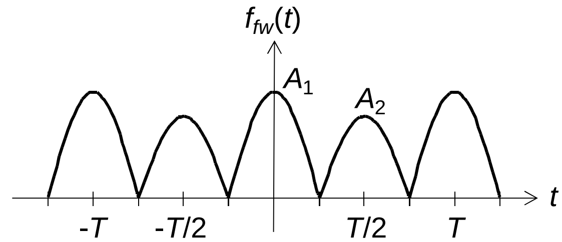
Homework 11

P9.1.9 Given a full-wave rectified waveform of period T, as shown in Figure 9.4.1b, except that because of dissymmetry in the rectifier circuit, the half-sinusoids are not all of the same amplitude but alternate with amplitudes of 12 V and 10 V. Derive the FSE, assuming that the sinusoid centered at the origin has a an amplitude of 12 V.



Solution P9.1.9

From Equation 9.4.1 the FSE of a half-wave rectified wave form of amplitude A_1 centered at the origin is: $\frac{A_1}{\pi}+\frac{A_1}{2}\cos\omega_0 t + \frac{2A_1}{\pi}\left(\frac{1}{3}\cos 2\omega_0 t - \frac{1}{15}\cos 4\omega_0 t + \frac{1}{35}\cos 6\omega_0 t + \ldots\right)$.

The FSE of a half-wave rectified waveform of amplitude A_2 delayed, or advanced, by $\frac{T}{2}$, where $\frac{\omega_0 T}{2} = \pi$, is: $\frac{A_2}{\pi}+\frac{A_2}{2}\cos(\omega_0 t \pm \pi) +$

$$\frac{2A_2}{\pi}\left[\frac{1}{3}\cos(2\omega_0 t \pm 2\pi) - \frac{1}{15}\cos(4\omega_0 t \pm 4\pi) + \frac{1}{35}\cos 6\omega_0 t + \ldots\right].$$

The sum of the two is: $\frac{A_1+A_2}{\pi}+\frac{A_1-A_2}{2}\cos\omega_0 t + \frac{2}{\pi}\left(\frac{A_1+A_2}{3}\cos 2\omega_0 t - \frac{A_1+A_2}{15}\cos 4\omega_0 t + \right.$

$\left.\frac{A_1+A_2}{35}\cos 6\omega_0 t + \ldots + \frac{(-1)^{n+1}}{4n^2-1}(A_1+A_2)\cos 6\omega_0 t + \ldots\right.$

Substituting $A_1 = 12$ and $A_2 = 10$, the terms of the FSE will have the following values:

dc term: $(A_1 + A_2)/\pi = 7.00$;

fundamental: $(A_1 - A_2)/2 = 1$;

2nd harmonic, $2(A_1 + A_2)/3\pi = 4.67$;

4th harmonic $2(A_1 + A_2)/15\pi = 0.934$;

6th harmonic $2(A_1 + A_2)/35\pi = 0.400$;

8th harmonic $2(A_1 + A_2)/63\pi = 0.222$;

10th harmonic $2(A_1 + A_2)/99\pi = 0.141$.

P9.1.10 Derive the FSE of the function shown in Figure 9.3.1.

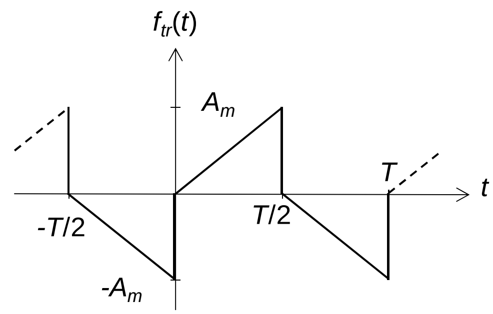


Figure 9.3.1

Solution P9.1.10

The function is half-wave symmetric and defined over half a period by $f(t) = \dfrac{A_m t}{T/2}$.

Hence, $C_n = 0$, for n even or zero. For n odd,

$$C_n = \frac{2}{T}\int_0^{T/2}\frac{A_m t}{T/2}e^{-jn\omega_0 t}dt = \frac{4A_m}{T^2}\int_0^{T/2} te^{-jn\omega_0 t}dt = \frac{4A_m}{T^2}\left[t\frac{e^{-jn\omega_0 t}}{-jn\omega_0} + \frac{e^{-jn\omega_0 t}}{n^2\omega_0^2}\right]_0^{\pi/\omega_0} =$$

$$\frac{4A_m}{T^2}\left[-\frac{j\pi}{n\omega_0^2} - \frac{2}{n^2\omega_0^2}\right] = -\frac{2A_m}{n^2\pi^2} - j\frac{A_m}{n\pi}.$$

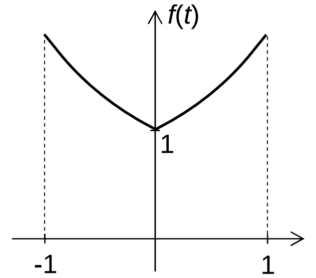
$a_n = -\dfrac{4A_m}{n^2\pi^2}$, and $b_n = \dfrac{2A_m}{n\pi}$.

The FSE is: $f(t) = \dfrac{2A_m}{\pi}\left[-\dfrac{2}{\pi}\sum_{n\text{ odd}}\dfrac{\cos n\omega_0 t}{n^2} + \sum_{n\text{ odd}}\dfrac{\sin n\omega_0 t}{n}\right].$

P9.1.11 A function is defined over half a period by $e^t, 0 < t < 1$. Derive the FSE if the function is: (a) even, (b) odd.

Solution P9.1.11

(a) The even function is as shown. $C_0 = a_0 = \frac{1}{1}\int_0^1 e^t dt = \left[e^t\right]_0^1 = e - 1$;

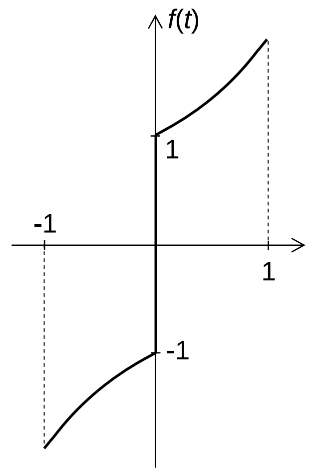


$$a_n = \frac{4}{2}\int_0^1 e^t \cos n\omega_0 dt = 2\mathrm{Re}[\int_0^1 e^{(1+jn\pi)t}dt] =$$

$$2\mathrm{Re}\left\{\frac{1}{1+jn\pi}\left[e^{(1+jn\pi)t}\right]_0^1\right\} = 2\mathrm{Re}\left\{\frac{e^{(1+jn\pi)}-1}{1+jn\pi}\right\};$$

it follows that $a_n = \frac{2(e\cos n\pi - 1)}{1+n^2\pi^2}$; $a_n = \frac{2(e-1)}{1+n^2\pi^2}$ for even n, and $a_n = -\frac{2(e+1)}{1+n^2\pi^2}$ for odd n.

(b) The odd function is as shown. $C_0 = a_0 = 0$. $b_n = 2\mathrm{Im}(\int_0^1 e^{(1+jn\pi)t}dt)$



$$= 2\mathrm{Im}\left\{\frac{1}{1+jn\pi}\left[e^{(1+jn\pi)t}\right]_0^1\right\} = 2\mathrm{Im}\left\{\frac{e^{(1+jn\pi)}-1}{1+jn\pi}\right\} =$$

$$2\mathrm{Im}\left\{\frac{e\cos n\pi - 1}{1+jn\pi}\right\}\ -\frac{2n\pi(e\cos n\pi - 1)}{1+n^2\pi^2};\ b_n = -\frac{2n\pi(e-1)}{1+n^2\pi^2}$$ for

even n, and $b_n = \frac{2n\pi(e+1)}{1+n^2\pi^2}$ for odd n.

P9.1.16 Derive the FSE of the waveform of Fig. P9.1.16 in three ways: (a) direct evaluation of coefficients; (b) shifting the waveform derived in P9.1.5; (c) from that of its derivative.

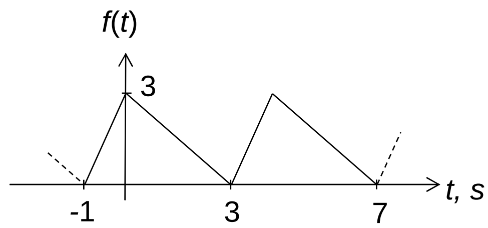


Figure P9.1.16

Solution P9.1.16

(a) $a_0 = C_0 = \frac{3\times 4}{2\times 4} = 1.5$; $C_n = \frac{1}{4}\int_{-1}^{0}(3t+3)e^{-jn\omega_0 t}dt$ +

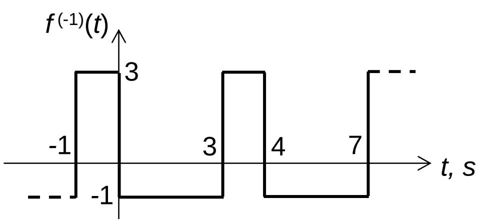
$$\frac{1}{4}\int_{0}^{3}(-t+3)e^{-jn\omega_0 t}dt = \frac{3}{4}\left[\frac{te^{-jn\omega_0 t}}{-jn\omega_0} + \frac{e^{-jn\omega_0 t}}{n^2\omega_0^2} + \frac{e^{-jn\omega_0 t}}{-jn\omega_0}\right]_{-1}^{0} + \frac{1}{4}\left[\frac{te^{-jn\omega_0 t}}{jn\omega_0} - \frac{e^{-jn\omega_0 t}}{n^2\omega_0^2} + \frac{3e^{-jn\omega_0 t}}{-jn\omega_0}\right]_{0}^{3} =$$

$$\frac{3}{4}\left[\frac{1}{n^2\omega_0^2} - \frac{e^{jn\pi/2}}{n^2\omega_0^2} + \frac{1}{-jn\omega_0}\right] + \frac{1}{4}\left[-\frac{e^{-j3n\pi/2}}{n^2\omega_0^2} + \frac{1}{n^2\omega_0^2} + \frac{3}{jn\omega_0}\right] = \frac{1}{n^2\omega_0^2}\left(1 - e^{jn\pi/2}\right) =$$

$$\frac{1}{n^2\omega_0^2}\left(1 - \cos\left(\frac{n\pi}{2}\right) - j\sin\left(\frac{n\pi}{2}\right)\right).$$

(b) From P9.1.5, $C_n = \frac{4}{n^2\pi^2}\left(e^{-jn\pi/2} - 1\right) = \frac{1}{n^2\omega_0^2}\left(e^{-jn\pi/2} - 1\right)$. If advanced by $T/4$, C_n is multiplied by $e^{jn\pi/2}$ and becomes $\frac{e^{jn\pi/2}}{n^2\omega_0^2}\left(e^{-jn\pi/2} - 1\right) = \frac{1}{n^2\omega_0^2}\left(1 - e^{jn\pi/2}\right)$, the same as in (a).

(c) When differentiated, the result is the rectangular pulse train shown, of zero average. Compared to the pulse train of Fig. 9.2.5, it has $A = 4$, $\alpha = 1/4$, and is advanced by 1/2 s. Hence, $C'_n = \frac{4}{n\pi}\sin\frac{n\pi}{4}e^{jn\pi/4}$.



The integral waveform has $C_n = \frac{C'_n}{jn\omega_0} = \frac{1}{n^2\omega_0^2}\frac{2}{j}\sin\frac{n\pi}{4}e^{in\pi/4} = \frac{e^{in\pi/4}}{n^2\omega_0^2}\left(e^{-jn\pi/4} - e^{in\pi/4}\right) =$

$\frac{1}{n^2\omega_0^2}\left(1 - e^{jn\pi/2}\right)$, as above.

The constant of integration is the average value of the integrated function.

P9.1.17 Obtain the FSE of a full-wave rectified waveform in two ways: (a) as the sum of two half-wave rectified waveforms, with one waveform shifted by half a period with respect to the other waveform; (b) as the product of a square wave of zero average and $\cos\omega_0 t$.

Solution P9.1.17

(a) $f_{hw}(t) = \frac{A}{\pi} + \frac{A}{2}\cos\omega_0 t + \frac{2A}{\pi}\left(\frac{1}{3}\cos 2\omega_0 t - \frac{1}{15}\cos 4\omega_0 t + \frac{1}{35}\cos 6\omega_0 t + \ldots + \frac{(-1)^{n+1}}{4n^2-1}\cos 2n\omega_0 t + \ldots\right)$

, n = 1, 2, 3, …

If delayed by $T/2$, the fundamental becomes: $\frac{A}{2}\cos(\omega_0 t - \pi) = -\frac{A}{2}\cos\omega_0 t$.

The nth term becomes $\frac{(-1)^{n+1}}{4n^2-1}\cos(2n\omega_0 t - 2n\pi) = \frac{(-1)^{n+1}}{4n^2-1}\cos 2n\omega_0 t$.

Adding the two functions gives Equation 9.4.2.

(b) If $A\cos\omega_0 t$ is multiplied by a square wave unit amplitude, zero average, centered at the origin, a full-wave rectified waveform is obtained.

From Equation 9.2.22, the FSE of the square waveform is:

$\frac{4A}{\pi}\left[\cos\omega_0 t - \frac{1}{3}\cos\omega_0 t + \frac{1}{5}\cos\omega_0 t - \frac{1}{7}\cos\omega_0 t + \ldots\right]$.

Multiplying by $A\cos\omega_0 t$ gives: $f_{fw}(t)$ =

$\frac{4A}{\pi}\left[\cos^2\omega_0 t - \frac{1}{3}\cos\omega_0 t\cos 3\omega_0 t + \frac{1}{5}\cos\omega_0 t\cos 5\omega_0 t - \frac{1}{7}\cos\omega_0 t + \ldots\right] =$

$\frac{2A}{\pi} + \frac{2A}{\pi}\left[(\cos 2\omega_0 t)\left(1 - \frac{1}{3}\right) + (\cos 4\omega_0 t)\left(-\frac{1}{3} + \frac{1}{5}\right) + (\cos 6\omega_0 t)\left(\frac{1}{5} - \frac{1}{7}\right) + \ldots\right] =$

$\frac{2A}{\pi} + \frac{4A}{\pi}\left[(\cos 2\omega_0 t)\left(\frac{1}{3}\right) + (\cos 4\omega_0 t)\left(-\frac{1}{15}\right) + (\cos 6\omega_0 t)\left(\frac{1}{35}\right) + \ldots\right]$, which is the same as Equation 9.4.2.