Homework 9

P7.1.4 An impedance $4+j4\ \Omega$ is connected in parallel with an impedance $12+j16\ \Omega$ If the total reactive power absorbed is 2500 VAR, what is the total real power absorbed? Verify your answer by working in terms of the overall impedance and admittance.

Solution P7.1.4

$Z = (4 + j4)(12 + j16)/(16 + j20) = 3.024 + j3.22\ \Omega$.

$Q = |\mathbf{I}|^2X$, and $P = |\mathbf{I}|^2R$, so that $P = 2500R/X = 2348$ W.

$Y = \dfrac{1}{4+j4} + \dfrac{1}{12+j16} = 0.155 - j0.165$ S; $Q = -|\mathbf{V}|^2B$, and $P = |\mathbf{V}|^2G$, so that $P = -2500G/B =$ 2348 W.

P7.1.5 Determine C in Figure P7.1.5 if the capacitor absorbs 5 VAR and the frequency is 50Hz. Derive the power absorbed by C from conservation of power in the circuit.

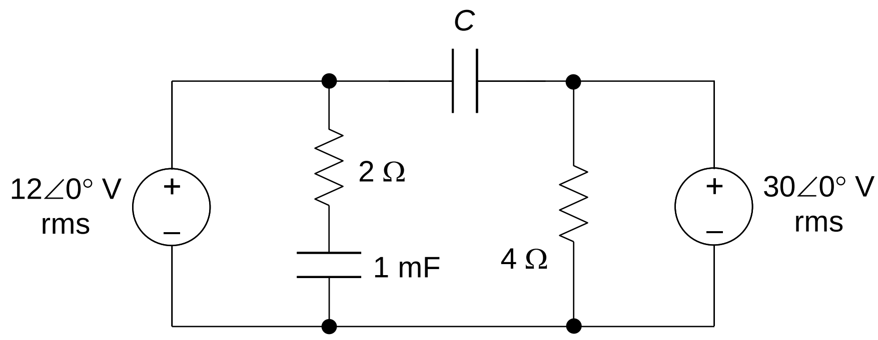
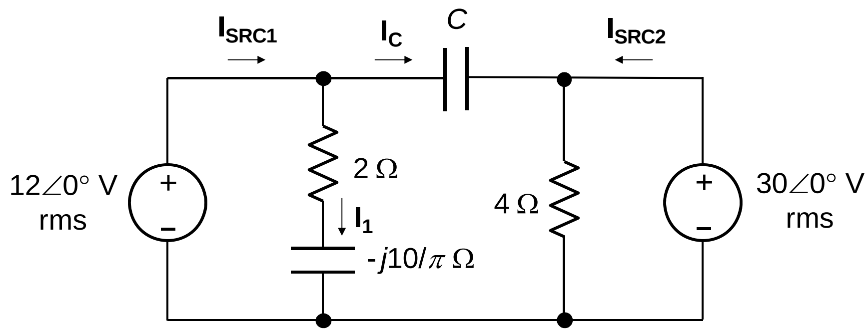


Figure P7.1.5

Solution P7.1.5



Voltage across C is 18 V. Hence, $\omega C|\mathbf{V}|^2 = 5$, which gives $C = 49.1\ \mu$F.

$\mathbf{I_C} = j\omega C(12 - 30) = -j5/18$ A.

For the 1 mF capacitor, $X = -1/(100\pi\times10^{-3}) = -10/\pi$; $\mathbf{I_1} = \dfrac{12}{2 + jX} = 1.70 + j2.70$ A.

Hence, $\mathbf{I_{SCR1}} = \mathbf{I_1} + \mathbf{I_C} = 1.70 + j\,2.43$ A;

$\mathbf{I_{SCR2}} = 7.5 - \mathbf{I_C} = 7.5 + j0.28$ A.

Complex power delivered by 12 V source: $\mathbf{S_{SRC1}} = 12\,I^*_{SRC1} = 20.38 - j29.1$ VA;

complex power delivered by 30 V source: $\mathbf{S_{SRC2}} = 30\,I^*_{SRC2} = 225 - j8.33$ VA;

complex power absorbed by capacitive branch: $\mathbf{S_1} = 12\,I^*_1 = 20.38 - j32.43$ VA;

power absorbed by resistive branch is 900/4 = 225 W.

Power absorbed by capacitor is $\mathbf{S_{SRC1}} + \mathbf{S_{SRC2}} - \mathbf{S_1} - 225 = -j5$ VAR.

P7.1.6 Determine R and the rms magnitude of $\mathbf{V_{SRC}}$ in Figure P7.1.6, given that each resistor absorbs 2 W and $\omega = 1{,}000$ rad/s.

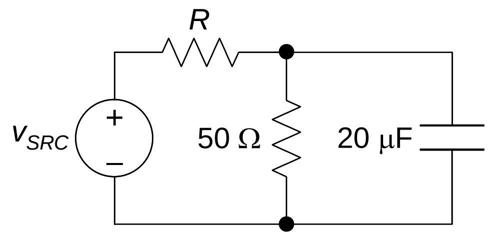
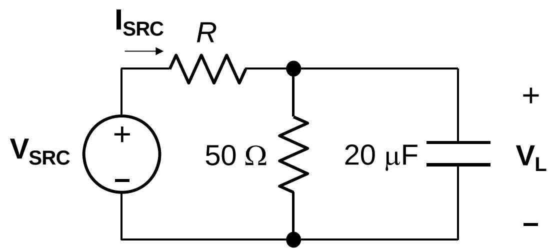


Figure P7.1.6

<u>Solution P7.1.6</u>

$$\frac{1}{\omega C} = \frac{1}{10^3 \times 20 \times 10^{-6}} = 50\ \Omega.$$



Since the 50 Ω resistor absorbs 2 W, $\frac{|V_L|^2}{50} = 2$,

$|\mathbf{V_L}| = 10$ V rms;

reactive power absorbed by capacitor is

$$-\frac{|V_L|^2}{50} = -2\ \text{VAR};$$

complex power absorbed by parallel capacitive branch is $2 - j2$ VA;

current through R is $\mathbf{I_{SRC}} = (2 + j2)/10 = 0.2 + j0.2 = 0.2\sqrt{2}\angle 45^\circ$ A rms.

Since the power dissipated in R is 2 W, $R = 2/(0.04\times 2) = 25\ \Omega$.

Complex power delivered by source is $4 - j2 = \mathbf{V_{SRC}}(0.2 - j0.2)$;

hence, $\mathbf{V_{SRC}} = \frac{4 - j2}{0.2 - j0.2} = 15 + j5 = 5\sqrt{10}\angle 18.4^\circ$ V rms.

P7.1.9 Determine the complex power delivered by $\mathbf{V}_{\mathbf{SRC}1}$ and $\mathbf{V}_{\mathbf{SRC2}}$ in Figure P7.1.9 given that L_1 absorbs 4 kW at a power factor of 0.6 lagging, L_2 absorbs 3 kW at a power factor of 0.6 leading, and the complex power absorbed by L_3 is 12 + *j*5 kVA. Assume that $\mathbf{V}_{\mathbf{SRC}1} = 400\angle 0^o$ V rms and $\mathbf{V}_{\mathbf{SRC}2} = 400\angle 90^o$ V rms.

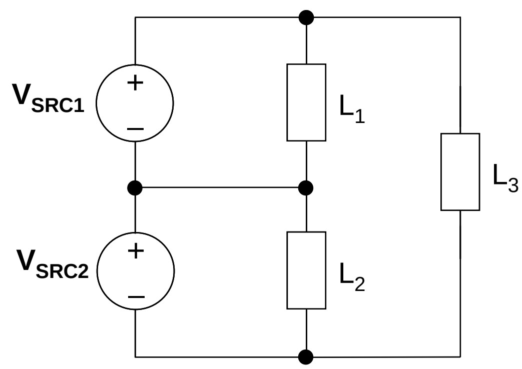


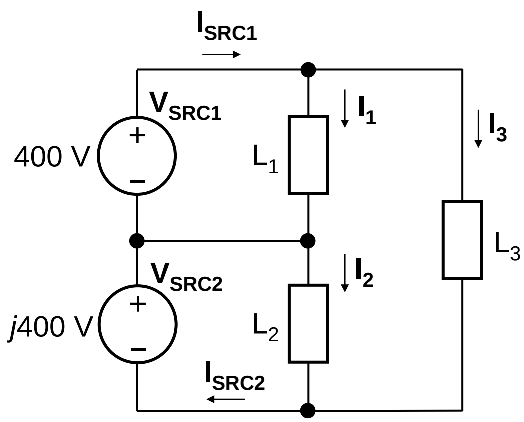
Figure P7.1.9

Solution P7.1.9

$S_1 = 4 + j\dfrac{4\times 0.8}{0.6} = 4 + j16/3$ kVA;

$S_2 = 3 - j\dfrac{3\times 0.8}{0.6} = 3 - j4$ kVA; $S_3 = 12 + j5$ kVA;

$S_1 = 400 I_1^*$, $S_2 = j400 I_2^*$, $S_3 = 400(1 + j) I_3^*$.



It follows that $\mathbf{I_1} = \dfrac{4 - j16/3}{400}\times 1000 =$

$10 - j40/3$ A;

$\mathbf{I_2} = \dfrac{3 + j4}{-j400}\times 1000 = -10 + j7.5$ A;

$\mathbf{I_3} = \dfrac{12 - j5}{400(1 - j)}\times 1000 = 21.25 + j8.75$ A;

$\mathbf{I_{SRC1}} = \mathbf{I_1} + \mathbf{I_3} = 31.25 - j4.58$;

$\mathbf{I_{SRC2}} = \mathbf{I_2} + \mathbf{I_3} = 11.25 + j16.25$ A;

hence, $S_{SRC1} = 400(31.25 + j4.58)/1000 = 12.5 + j1.83$ kVA;

$S_{SRC2} = j400(11.25 - j16.25)/1000 = 6.5 + j4.5$ kVA.

As a check, $S_{SRC1} + S_{SRC2} = 19 + j19/3 = S_1 + S_2 + S_3$.

P7.1.11 In Figure P7.1.11, find the instantaneous power, the real power, and reactive power delivered by the source, given that $v_{SRC} = 10\cos 10^6 t$.

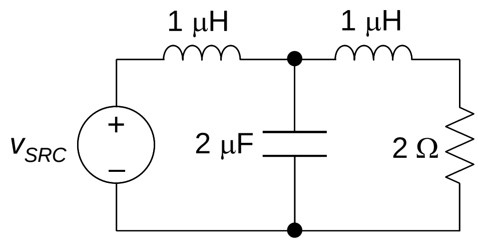


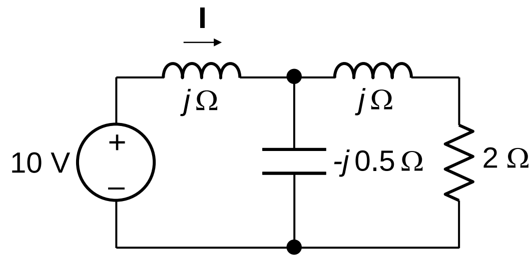
Figure P7.1.11

<u>Solution P7.1.11</u>

ωL = 1 Ω; $1/\omega C$ = 0.5 Ω;

$\mathbf{I} = \dfrac{10}{j + (-j * 0.5) \,||\, (j+2)} = 5 - j20 = 20.62\angle\text{-}76° \equiv$

$20.62\cos(10^6 t - 76°)$ A.



The instantaneous power is $206.2(\cos 10^6 t)\cos(10^6 t - 76°) = 103.1[\cos(76°) + \cos(2\times 10^6 t - 76°)]$ VA.

The complex power is $10(5 + j20)/2 = 103.1\angle 76°$ VA in terms of rms values, so that the real power is $103.1\cos(76°) = 25$ W and the reactive power is $103.1\sin(76°) = 100$ VAR.

P7.2.9 Determine Z_L in Figure P7.2.9 that makes it absorb maximum power and calculate this power.

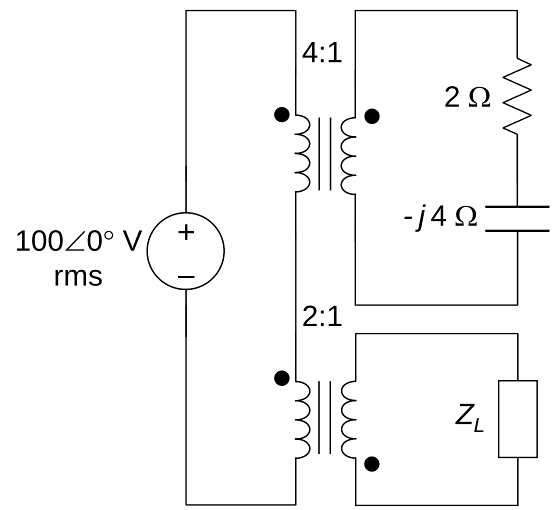
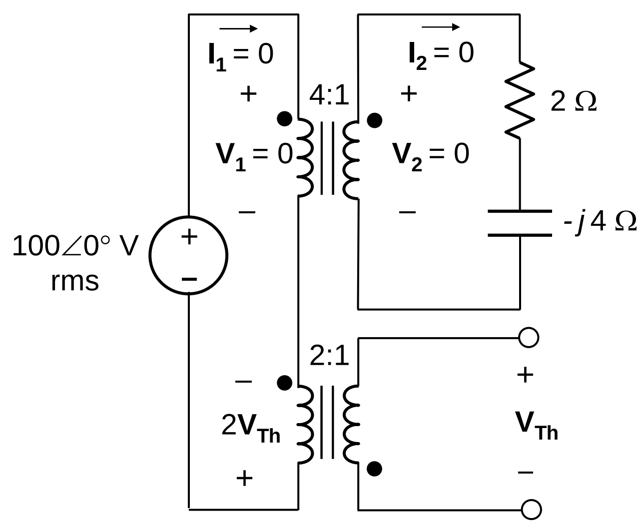


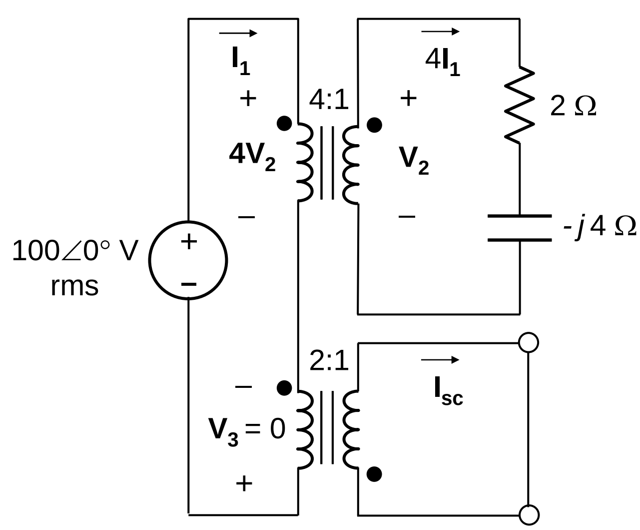
Figure P7.2.9

<u>Solution P7.2.9</u>

On open circuit, $\mathbf{I_1} = 0$, so that $\mathbf{I_2} = 0$, $\mathbf{V_2} = 0$, and $\mathbf{V_1} = 0$. It follows that $2\mathbf{V_{Th}} = -100$ V, and $\mathbf{V_{Th}} = -50$ V.



On short circuit, $\mathbf{V_3} = 0$; hence, $4\mathbf{V_2} = 100$, and $\mathbf{V_2} = 25$ V; $4\mathbf{I_1} = \dfrac{25}{2-j4}$, and $\mathbf{I_{sc}} = -2\mathbf{I_1} = \dfrac{-25}{4(1-j2)}$. Hence $Z_{Th} = 8(1 - j2)$ Ω.



It follows that for maximum power transfer, $Z_{Lm} = 8(1 + j2)$ Ω. $|\mathbf{I_{Lm}}| = 50/16$ A, and $R_{Lm}|\mathbf{I_{Lm}}|^2 = 8(50/16)^2 = 78.125$ W.

P7.2.13 Determine Z_L in Figure P7.2.13 that will absorb maximum power and calculate this power.

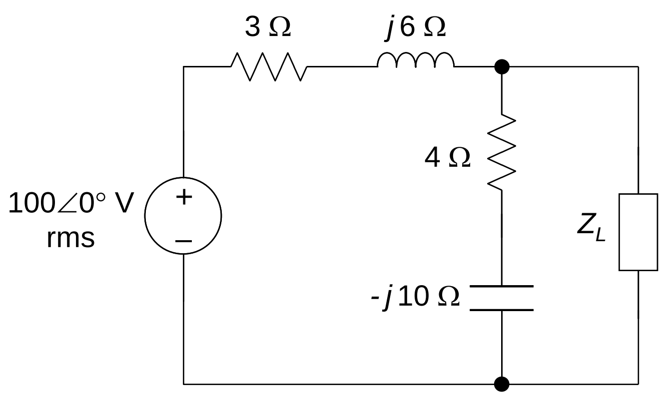


Figure P7.2.13

Solution P7.2.13

If Z_L is removed, the open-circuit voltage is: $\mathbf{V_{Th}} = \frac{4 - j10}{7 - j4} \times 100 = \frac{68 - j54}{65} \times 100$ = 104.6 – j83.08 = 133.6∠-38.5° V.

If the voltage source is set to zero, $Z_{Th} = \frac{(3 + j6)(4 - j10)}{7 - j4}$ = 8.123 + j3.785 Ω.

Hence for maximum power transfer, Z_{Lm} = 8.123 – j3.785 Ω.

The maximum power transferred is $(V_{Th})^2/4R_{Lm} = (133.6)^2/(4\times8.123)$ = 549.2 W.