Homework 2

P1.3.6 The triangular voltage waveform of Figure P1.3.6 is applied to a 100 Ω resistor. Determine: (a) the resistor current, (b) instantaneous power dissipation in the resistor, and (c) the average power dissipation. Is the statement 'the average power in a resistor is the product of the average voltage across the resistor and the average current through the resistor' valid? Justify your answer.

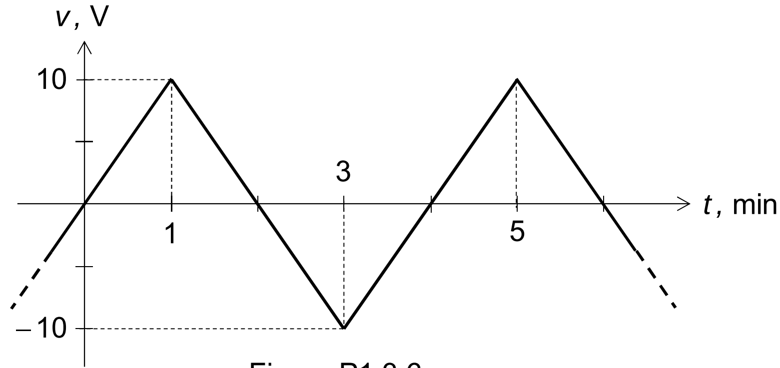


Figure P1.3.6

Solution P1.3.6

$v = 10t$ V, $0 \le t \le 1$ min;

$v = -10t + 20$ V, $1 \le t \le 3$ min;

$v = 10t - 40$ V, $3 \le t \le 4$ min.

(a) $i = \dfrac{v}{100} = 0.1t$ A, $0 \le t \le 1$ min;

$i = -0.1t + 0.2$ A, $1 \le t \le 3$ min;

$i = 0.1t - 0.4$ A, $3 \le t \le 4$ min.

(b) $p = \dfrac{v^2}{R} = t^2$ W, $0 \le t \le 1$ min;

$$p = \frac{(-10t + 20)^2}{100} = (t-2)^2 \text{ W}, \ 1 \le t \le 3 \text{ min};$$

$$p = \frac{(10t - 40)^2}{100} = (t-4)^2 \text{ W}, \ 3 \le t \le 4 \text{ min}.$$

(c) $$P = \frac{1}{T}\int_0^T p\,dt = \frac{1}{4}\left[\int_0^1 t^2 dt + \int_1^3 \left(t^2 - 4t + 4\right)dt + \int_3^4 \left(t^2 - 8t + 16\right)dt\right]$$

$$= \frac{1}{4}\left\{\left[\frac{t^3}{3}\right]_0^1 + \left[\frac{t^3}{3} - 2t^2 + 4t\right]_1^3 + \left[\frac{t^3}{3} - 4t^2 + 16t\right]_3^4\right\}$$

$$= \frac{1}{4}\left\{\frac{1}{3} + \frac{2}{3} + \frac{1}{3}\right\} = \frac{1}{3} \text{ W}.$$

The following should be noted:

1. The integral is in V×I×min. When divided by T in min, the result is in W.
2. The area under each quarter of a cycle is the same, which means that the average is the same. Hence the average can be determined from the first quarter cycle:

$$P = \frac{1}{1}\int_0^1 t^2 dt = \left[\frac{t^3}{3}\right]_0^1 = \frac{1}{3} \text{ W.}$$

3. If t is converted to seconds, then v during the first quarter cycle is $v = \frac{10t}{60} = \frac{t}{6}$ V,

$i = \frac{t}{600}$ A, $p = \frac{t^2}{3600}$ $0 \le t \le 60$ s;

$$P = \frac{1}{60}\int_0^{60} \frac{t^2}{3600} dt = \frac{1}{60^3}\left[\frac{t^3}{3}\right]_0^{60} = \frac{1}{3} \text{ W.}$$

(d) V_{avg} = 0, since the waveform is symmetrical about the horizontal axis. This makes I_{avg} = 0 as well. Thus, $V_{avg} \times I_{avg}$ = 0, whereas $P \neq 0$. The average power in a resistor is not the product of the average voltage across the resistor and the average current through the resistor, because power, being the product of voltage and current, is a nonlinear quantity. The product of a negative voltage and a negative current is positive power, which contributes to te average power

P1.3.9 The voltage shown in Figure P1.3.9 is applied across a 5 Ω resistor. (a) Determine p, $0 \le t \le 1\,\text{min}$; (b) the energy dissipated in the resistor at t = 3 min.

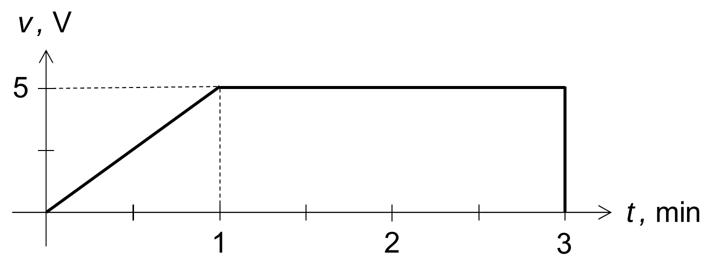


Figure P1.3.9

<u>Solution P1.3.9</u>

$$v\ (\text{V}) = \begin{cases} t/12 & 0 \le t \le 60\ \text{s} \\ 5 & 60 \le t < 180\ \text{s} \\ 0 & t > 180\ \text{s} \end{cases}$$

(a) $0 \le t \le 60\ \text{s};\ p = \dfrac{v^2}{R} = \dfrac{1}{5}\left(\dfrac{t}{12}\right)^2 = \dfrac{t^2}{720}$ W, where t is in s.

(b) $w = \int_0^{180} p dt = \int_0^{60} \dfrac{t^2}{720} dt + \int_{60}^{180} \dfrac{25}{5} dt = 700\ \text{J}$.

P1.4.12 The triangular voltage pulse of Figure P1.4.12 is applied to a $0.1\mu\text{F}$ capacitor that is initially uncharged. Plot as function of time: (a) the charge on the capacitor; (b) the energy stored in the capacitor, as derived from Equation 1.8.4, (c) the capacitor current; (d) the instantaneous power input to the capacitor. How is the energy stored in (b) related to the instantaneous power in (d)?

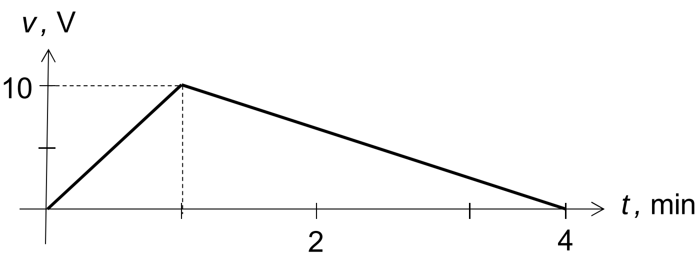


Figure P1.4.12

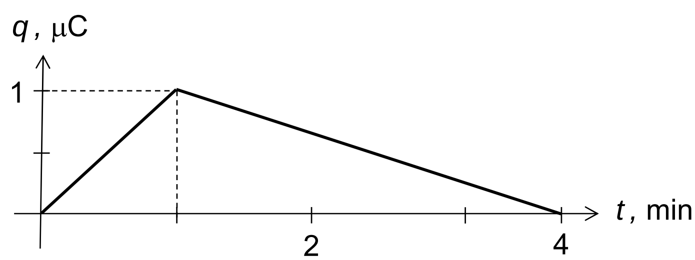
Solution P1.4.12

$v = \dfrac{t}{6}\text{V},\ 0 \le t \le 60\text{ s};\ v = -\dfrac{t}{18} + \dfrac{40}{3}\text{V},\ 60 \le t \le 240\text{ s};\ t = 0,\ t \ge 240\text{ s}.$

(a) $q = Cv = \dfrac{t}{60}\,\mu\text{C},\ 0 \le t \le 60\text{ s};$

$= -\dfrac{t}{180} + \dfrac{4}{3}\,\mu\text{C},\ 60 \le t \le 240\text{ s};$

$= 0,\ t \ge 240\text{ s}.$

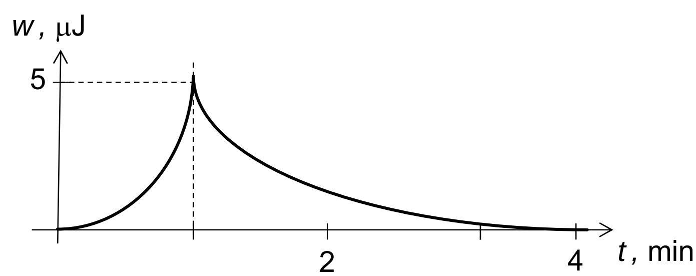


(b) $w = \dfrac{1}{2}qv = \dfrac{t^2}{720}\,\mu\text{J},\ 0 \le t \le 60\text{ s};$

$= \dfrac{1}{180}\left(\dfrac{t^2}{36} - \dfrac{40t}{3} + 1600\right)\mu\text{J},$

$60 \le t \le 240\text{ s};$

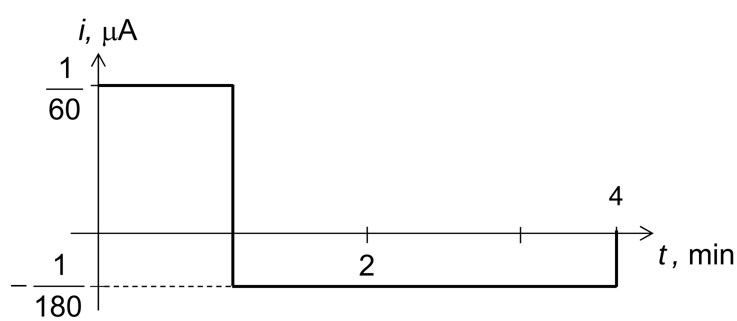
$= 0,\ t \ge 240\text{ s}.$



(c) $i = \dfrac{dq}{dt} = \dfrac{1}{60}\,\mu\text{A},\ 0 < t < 60\text{ s};$

$= -\dfrac{1}{180}\,\mu\text{A},\ 60 < t < 240\text{ s};$

$= 0,\ t > 240\text{ s}$, where $\dfrac{1}{60}\,\mu\text{A}$ may also be expressed as $1\ \mu\text{C/min}$.

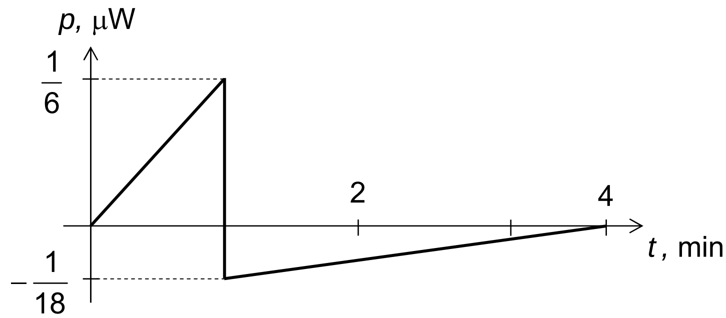


(d) $p = vi = \frac{t}{360}\,\mu\text{W},\ 0 \le t \le 60\text{ s};$

$= \frac{1}{180}\left(\frac{t}{18} - \frac{40}{3}\right)\mu\text{W},$

$60 \le t \le 240\text{ s};$

$= 0,\ t \ge 240\text{ s},$ where $\frac{1}{6}\,\mu\text{W}$



may also be expressed as 10 μJ/min.

It is seen that $w = \int p\,dt$. Thus $\int_0^t \frac{t}{360}dt = \frac{t^2}{720}\,\mu\text{J},\ 0 \le t \le 60\text{ s}$. At t = 60 s, w = 5 μJ.

For $60 \le t \le 240\text{ s}$ $w = \int_{60}^t \frac{1}{180}\left(\frac{t}{18} - \frac{40}{3}\right)dt + 5 = \frac{1}{180}\left(\frac{t^2}{36} - \frac{40t}{3} + 1600\right)\mu\text{J}.$

P1.5.9 A triangular current waveform is described by Figure P1.4.12, with the current in amperes replacing the voltage in volts. The triangular current pulse is applied to a $0.1\mu H$ inductor that has no initial energy storage. Plot as function of time: (a) the flux linkage in the inductor; (b) the energy stored in the inductor, as derived from Equation 1.9.10; (c) the voltage across the inductor; (d) the instantaneous power input to the inductor. How is the energy stored in (b) related to the instantaneous power in (d)?

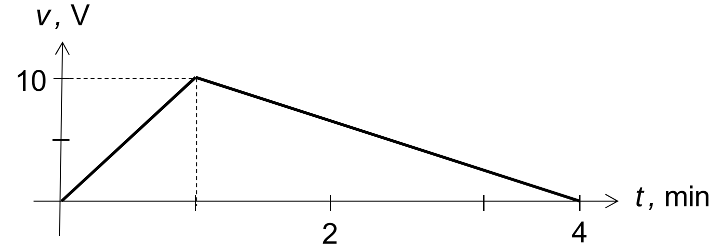


Figure P1.4.12

Solution P1.5.9

$i = \dfrac{t}{6}$ A, $0 \le t \le 60$ s;

$i = -\dfrac{t}{18} + \dfrac{40}{3}$ A, $60 \le t \le 240$ s;

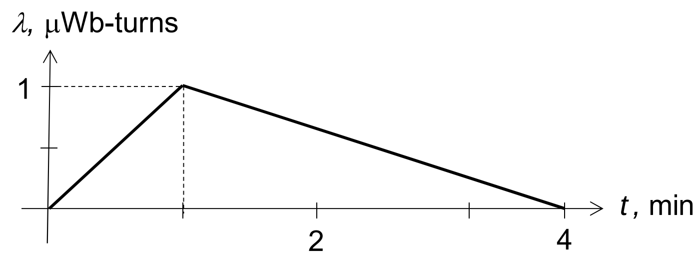
$i = 0$, $t \ge 240$ s.

(a) $\lambda = Li = \dfrac{t}{60}$ μWb-turns,

$0 \le t \le 60$ s;

$= -\dfrac{t}{180} + \dfrac{4}{3}$ μWb-turns,

$60 \le t \le 240$ s $= 0$, $t \ge 240$ s.

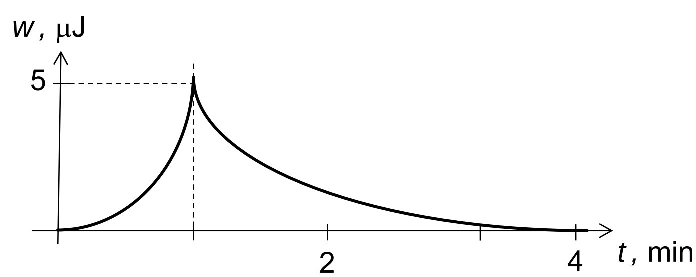


(b) $w = \dfrac{1}{2}\lambda i = \dfrac{t^2}{720}$ μJ, $0 \le t \le 60$ s;

$= \dfrac{1}{180}\left(\dfrac{t^2}{36} - \dfrac{40t}{3} + 1600\right)$ μJ,

$60 \le t \le 240$ s;

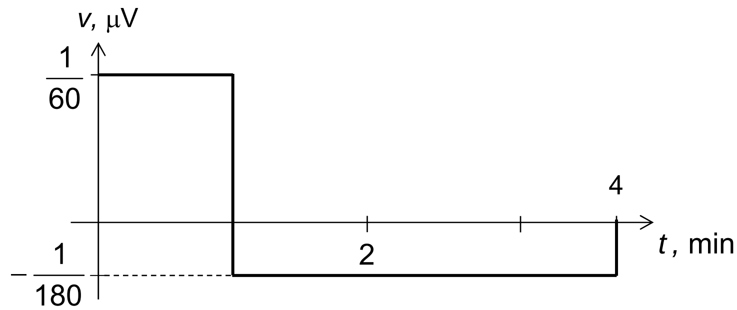
$= 0$, $t \ge 240$ s.



(c) $v = \dfrac{d\lambda}{dt} = \dfrac{1}{60}\ \mu\text{V},\ 0 < t < 60\ \text{s};$

$= -\dfrac{1}{180}\ \mu\text{v},\ 60 < t < 240\ \text{s};$

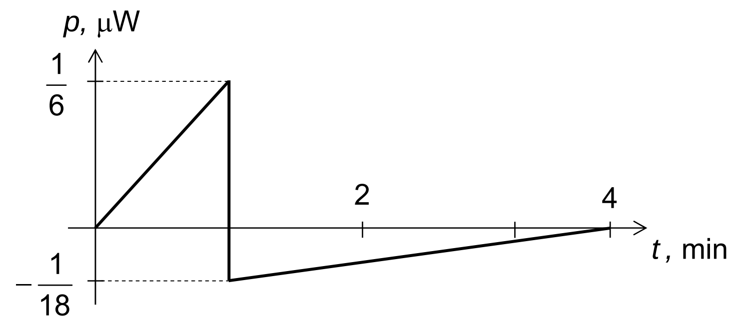
$= 0,\ t > 240\ \text{s}$, where $\dfrac{1}{60}\ \mu\text{V}$ may also be expressed as 1 μWb-turns/min.



(d) $p = vi = \dfrac{t}{360}\ \mu\text{W},\ 0 \le t \le 60\ \text{s};$

$= \dfrac{1}{180}\left(\dfrac{t}{18} - \dfrac{40}{3}\right)\ \mu\text{W},$ $60 \le t \le 240\ \text{s};$

$= 0,\ t \ge 240\ \text{s}$, where $\dfrac{1}{6}\ \mu\text{W}$ may also be expressed as 10 μJ/min.



It is seen that $w = \int p dt$. Thus $\int_0^t \dfrac{t}{360} dt = \dfrac{t^2}{720}\ \mu\text{J},\ 0 \le t \le 60\ \text{s}$. At t = 60 s, w = 5 μJ.

For $60 \le t \le 240\ \text{s}$ $w = \int_{60}^t \dfrac{1}{180}\left(\dfrac{t}{18} - \dfrac{40}{3}\right)dt + 5 = \dfrac{1}{180}\left(\dfrac{t^2}{36} - \dfrac{40t}{3} + 1600\right)\mu\text{J}.$

Note that the numerical values in this problem are identical to those of P1.4.12, if v and i, and q and λ, are interchanged.

P2.1.4 Determine the voltage across each current source and the current through each voltage source in Figure P2.1.4.

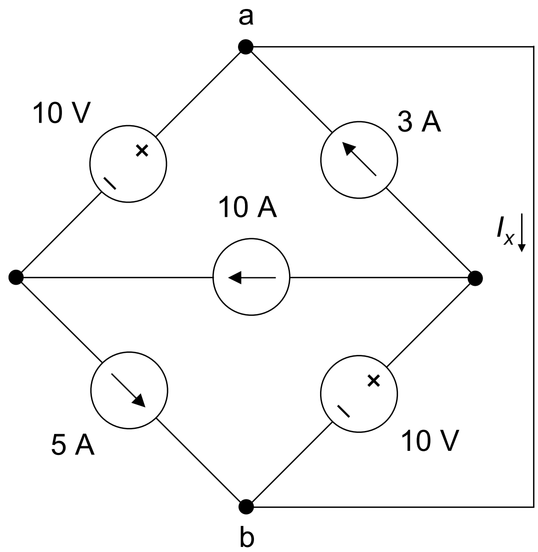
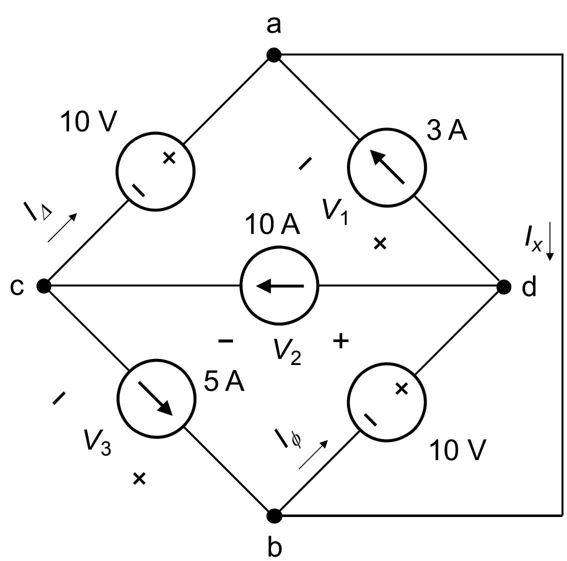


Figure P2.1.4

<u>Solution P2.1.4</u>



From KCL at node c: $10 = 5 + I_\Delta$, so $I_\Delta = 5$ A.

From KCL at node d: $I_\phi = 10 + 3 = 13$ A.

From KCL at node a: $I_x = 3 + I_\Delta = 8$ A.

As a check, KCL at node b: $I_x + 5 = I_\phi$, or $8 + 5 = 13$.

From KVL around the outer loop, starting from node a: $-V_3 + 10 = 0$, or $V_3 = 10$ V.

From KVL around the mesh on the RHS, starting from node a: $+10 - V_1 = 0$, or $V_1 = 10$ V.

From KVL around the mesh cdb, starting from node c: $V_2 - 10 - V_3 = 0$. This gives: $V_2 = 20$ V.

As a check, KVL around the mesh adc, staring frpm node a: $V_1 - V_2 + 10 = 0$, or $10 - 20 + 10 = 0$.

P2.2.5 Determine G_{eq} between terminals ab in Figure P2.2.5.

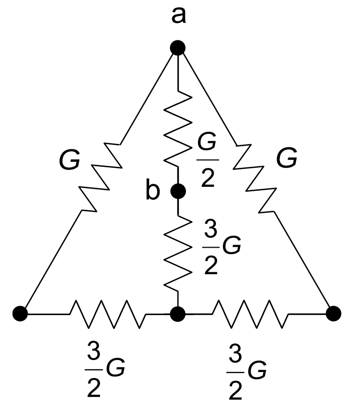


Figure P2.2.5

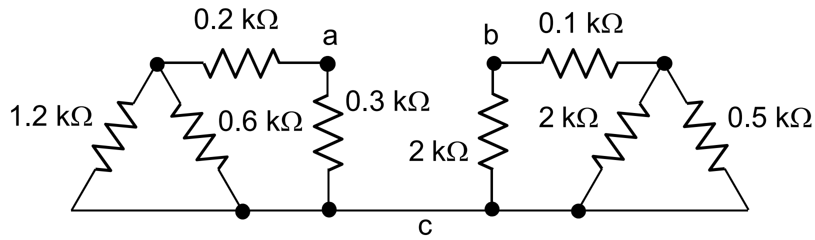
Solution P2.2.5

G in series with $\frac{3}{2}G$ is $\frac{1\times 1.5}{2.5}$ =0.6 G.

The two 0.6 G resistors ae in parallel, giving a total conductance of 1.2 G. This conductance is in series with 1.5 G, which gives

$\frac{1.5\times 1.2}{1.5+1.2}G=\frac{2}{3}G$; hence, $G_{ab}=\frac{G}{2}+\frac{2G}{3}=\frac{7}{6}G$.

P2.2.6 Determine R_{eq} between terminals ab in Figure P2.2.6.



Solution P2.2.6

Starting with the RHS, (2 kΩ)||(0.5 kΩ) = $\frac{2 \times 0.5}{2.5} = 0.4\,\text{k}\Omega$, 0.1 + 0.4 = 0.5 kΩ,

R_{bc} = (2 kΩ)||(0.5 kΩ) = 0.4 kΩ;

on the LHS, (1.2 kΩ)||(0.6 kΩ) = $\frac{1.2 \times 0.6}{1.8} = 0.4$ kΩ, 0.2 + 0.4 = 0.6 kΩ,

R_{ac} = (0.3 kΩ)||(0.6 kΩ) = $\frac{0.3 \times 0.6}{0.9} = 0.2\,\text{k}\Omega$;

R_{ab} = 0.4 + 0.2 = 0.6 kΩ.