Homework 6

P5.1.6 Given the sinusoidal time function v of Figure P5.1.6. Express v as a function of time and as a phasor.

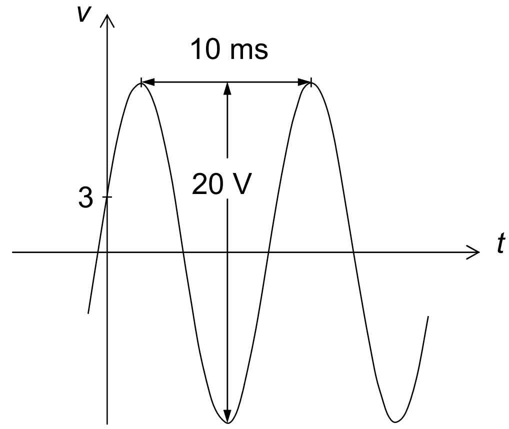


Figure P5.1.6

Solution P5.1.6

$v = 10\cos(\omega t + \varphi)$ V.

$\omega = 2\pi/T = 2\pi/(10*10^{-3}) = 200\pi$.

At $t = 0$, $v = 3$, so that $\cos(\varphi) = 0.3$, or, $\varphi = 72.54°$.

As a time function, $v = 10\cos(200\pi t + 72.54°)$ V.

As a phasor, $\mathbf{V} = 10\angle 72.54°$ V.

P5.2.1 Given $i = 8\sqrt{2}\cos\left(2{,}500\pi t - 45^\circ\right)$ A and i_1 = $2\cos 2{,}500\pi t$ A in Figure P5.2.1. Determine: (a) v and i_2 in the time and frequency domains; (b) Z, if composed of i) two series elements, or ii) two parallel elements.

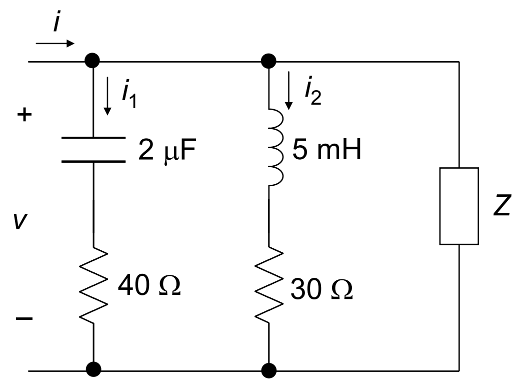


Figure P5.2.1

Solution P5.2.1

At $\omega = 2{,}500\pi$ rad/s

$R - j/\omega C = 40 - j/(2500\pi\times2\times10^{-6}) = 40 - j63.7 = 75.21\angle{-57.87^\circ}\ \Omega$.

(a) $\mathbf{V} = \mathbf{I_1}(75.21\angle{-57.87^\circ}) = (2\angle 0^\circ)\times 75.21\angle{-57.87^\circ}$ V $= 150.43\angle{-57.87^\circ}$ V.

$v = 150.43\cos(2500\pi t - 57.87)$ V.

$\mathbf{I_2} = \mathbf{V}/(30 + j\omega L) = (150.43\angle{-57.87^\circ})/(30 + j2500\pi\times5\times10^{-3})$ A $= (150.43\angle{-57.87^\circ})/(49.718\angle 52.62^\circ) = 3.04\angle{-110.49^\circ} = -1.065 - j2.85$.

$i_2 = 3.04\cos(2500\pi t - 110.49^\circ)$ A.

(b) $Z = \mathbf{V}/(\mathbf{I} - \mathbf{I_1} - \mathbf{I_2}) = (150.43\angle{-57.87^\circ})/(8 - j8 - 2 + 1.065 + j2.85)\ \Omega = (150.43\angle{-57.87^\circ})/(7.065 - j5.15)\ \Omega = (150.43\angle{-57.87^\circ})/(8.74\angle{-36.1^\circ})\ \Omega = 17.21\angle{-21.78^\circ} = 15.98 - j6.39\ \Omega$.

(i) If Z is composed of two series elements R and C, $R = 15.98\ \Omega$ and $C = 1/(2500\pi\times6.39) = 19.93$ μF.

(ii) If Z is composed of two parallel elements, $Y = \dfrac{1}{17.21\angle{-21.78^\circ}} = 0.053 + j0.021$ S $= G + j\omega C$, where $G = 0.053$ S and $C = 0.021/2500\pi = 2.67$ μF.

P5.2.5 Determine the input impedance Z_i in Figure P5.2.5.

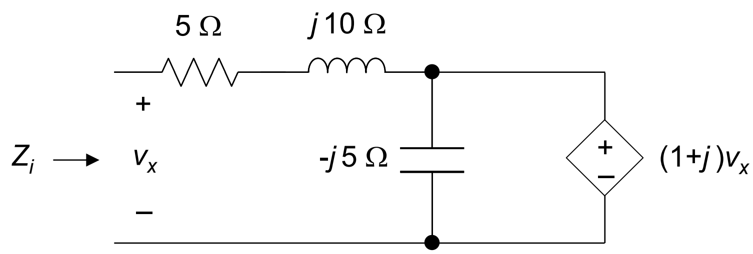


Figure P5.2.5

Solution P5.2.5

If a source v_X is applied, the source current being i_x, then: $v_x = 5i_x + j10i_x + (1+j)\, v_x$.

$$\frac{v_x}{i_x} = \frac{5 + j10}{-j} = -10 + j5\ \Omega.$$

Alternatively, the capacitor may be removed because it is in parallel with a voltage source. The current i_x is $\frac{v_x - (1+j)v_x}{5 + j10} = \frac{-jv_x}{5 + j10}$. This gives $\frac{v_x}{i_x} = \frac{5 + j10}{-j} = -10 + j5\ \Omega$, as before.

P5.3.32 Determine $\mathbf{I_1}$, $\mathbf{I_2}$, and V_{ab} in Figure P5.3.32, given that $\mathbf{V_{SRC}} = 50\angle 60^\circ$ V. Draw a phasor diagram showing these quantities.

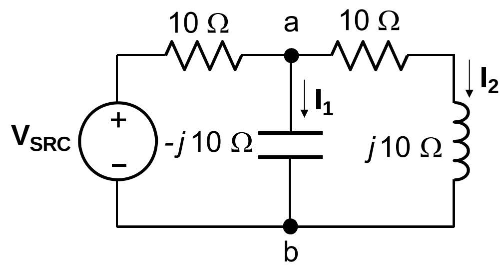


Figure P5.3.32

Solution P5.3.32

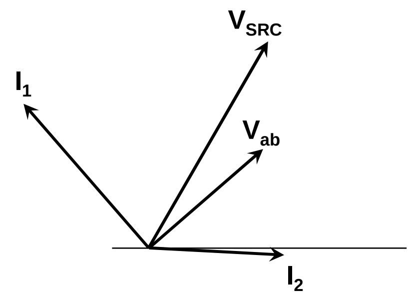
Taking node b as a reference, KCL at node a gives:

$$\frac{1}{-j10}\mathbf{V_a} + \frac{1}{10+j10}\mathbf{V_a} + \frac{1}{10}\mathbf{V_a} = 5\angle 60^\circ.$$

This gives $\mathbf{V_a} = \mathbf{V_{ab}} = 31.63\angle 41.6^\circ$ V.

$\mathbf{I_1} = \dfrac{1}{-j10}\mathbf{V_a} = 3.16\angle 131.6^\circ$ A,

and $\mathbf{I_2} = \dfrac{1}{10+j10}\mathbf{V_a} = 2.24\angle -3.43^\circ$ A.



The phasor diagram is shown, where the voltage and current scales are different.

P5.3.35 Figure P5.3.35 shows a **Maxwell bridge** that may be used for measuring the inductance and resistance of a coil in terms of known capacitance and resistance values. Show that at bridge balance: $R_2 = \frac{R_1 R_3}{R_4}$ and $L_2 = C_4 R_1 R_3$. Note that the first condition is a DC balance condition, the same as in the Wheatstone bridge.

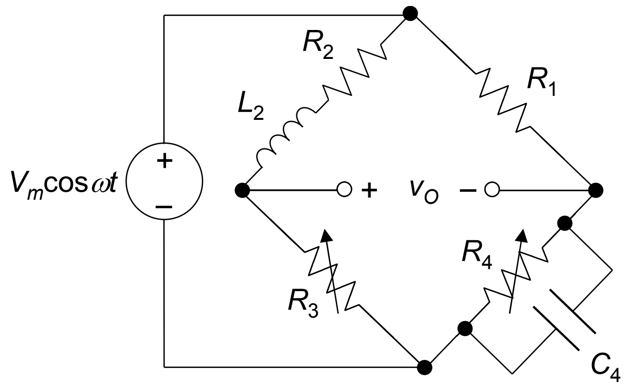


Figure P5.3.35

<u>**Solution P5.3.35**</u>

The impedance of arm 4 is: $\frac{R_4}{1 + j\omega C_4 R_4}$. At bridge balance: $\frac{R_2 + j\omega L_2}{R_3} = \frac{R_1(1 + j\omega R_4 C_4)}{R_4}$.

Equating real parts: $\frac{R_2}{R_3} = \frac{R_1}{R_4}$.

Equating imaginary parts: $\frac{L_2}{R_3} = C_4 R_1$.