Homework 7

P5.3.37 Determine NEC between terminals ab in Figure P5.3.37.

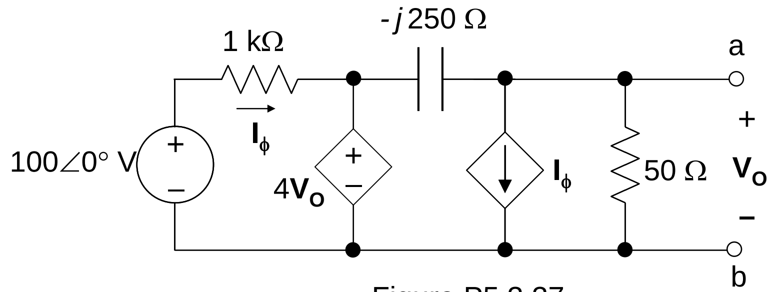


Figure P5.3.37

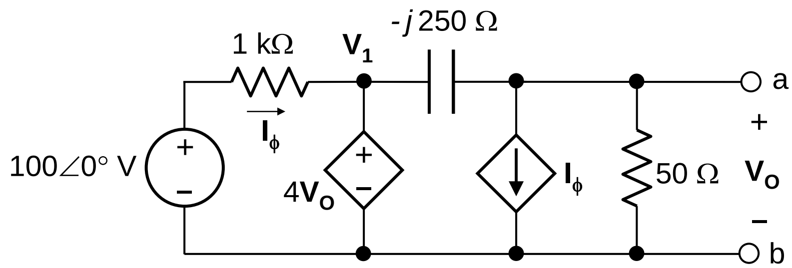
Solution P5.3.37

$\mathbf{V_1} = 4\mathbf{V_O}$; KCL at the right node:

$$-\frac{1}{j250}(\mathbf{V_O} - 4\mathbf{V_O}) + \mathbf{I_\phi} + \frac{1}{50}\mathbf{V_O} = 0,$$

where $\mathbf{I_\phi} = \dfrac{100 - \mathbf{V_1}}{1000}$. Substituting,

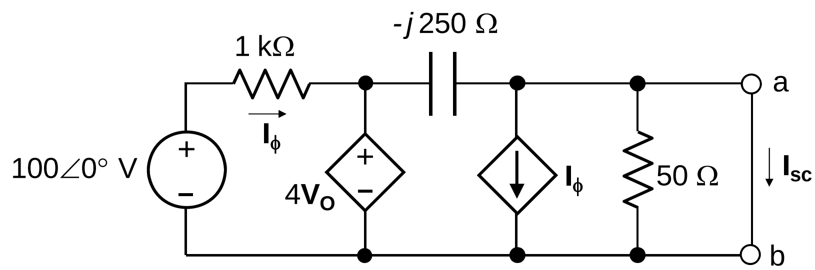
$$\frac{3}{j250}\mathbf{V_O} + \frac{100 - 4\mathbf{V_0}}{1000} + \frac{1}{50}\mathbf{V_O} = 0.$$



This gives $\mathbf{V_O} = -4 - j3$ V.

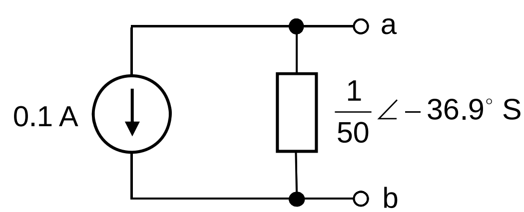
When terminals ab are short-circuited, both nodes are at zero voltage. It follows that

$\mathbf{I_\phi} = 0.1$ A and $\mathbf{I_{sc}} = \mathbf{I_N} = -0.1$ A.



Moreover, $Y_N = \dfrac{0.1}{4 + j3} = \dfrac{1}{250}(4 - j3) = \dfrac{1}{50}\angle\text{-}36.9°$ S.

NEC will then be as shown.



P5.3.38 Determine TEC between terminals ab in Figure P5.3.38.

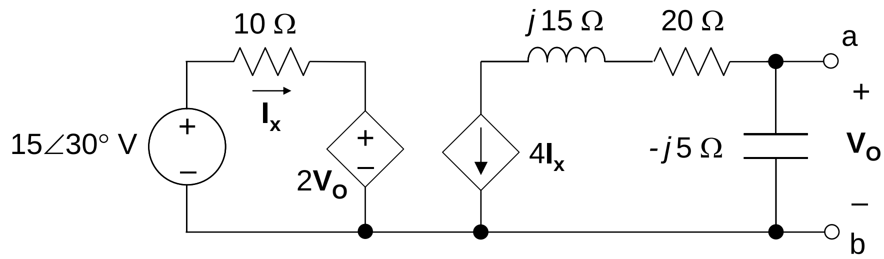


Figure P5.3.38

Solution P5.3.38

On open-circuit, $\mathbf{I_x} = \dfrac{15\angle 30° - 2\mathbf{V_O}}{10}$;

$\mathbf{V_O} = (-4\mathbf{I_x}) \times (-j5) = j20\mathbf{I_x} = j2(15\angle 30^\circ - 2\mathbf{V_O})$, or $\mathbf{V_O}(4 - j) = 30\angle 30^\circ$

$\mathbf{V_{Th}} = \mathbf{V_O} = \dfrac{30\angle 30°}{4 - j} = \dfrac{15}{17}\left[\left(4\sqrt{3} - 1\right) + j\left(4 + \sqrt{3}\right)\right] = 7.277\angle 44.0°$ V.

On short-circuit, $\mathbf{I_x} = \dfrac{15\angle 30°}{10}$ and $\mathbf{I_{sc}} = -4\mathbf{I_x} = -6\angle 30°$ A.

Hence, $Z_{Th} = -\dfrac{5}{4 - j} = \dfrac{-20}{17} - j\dfrac{5}{17}\,\Omega$.

P6.1.1 The terminal of one coil in Figure P6.1.1 is marked with a dot. Mark one terminal of the other coils with a dot and connect the coils in series for maximum total inductance.

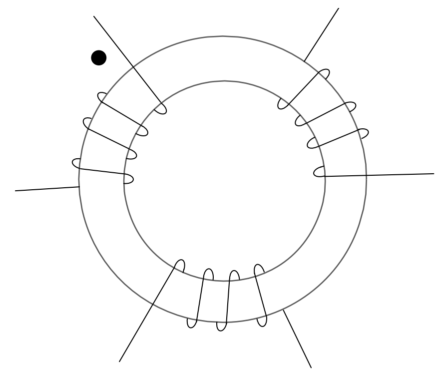
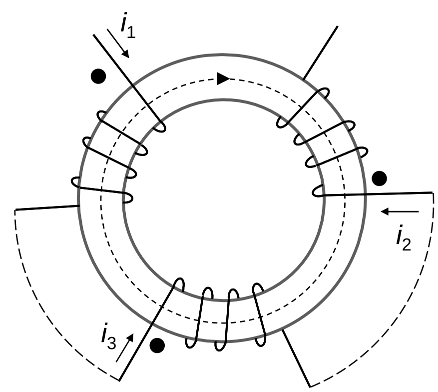


Figure P6.1.1

Solution P6.1.1



The dots on the windings should be such that current entering at the dotted terminals produces flux in the same direction in the core. To produce maximum inductance, the coils have to be connected so that fluxes are additive.

P6.1.2 Two coils are coupled through a high-permeability core. When i_1 = 4 A, ϕ_{11e} = 0.1 mWb and ϕ_{21} = 0.4 mWb. When i_2 = 3 A, ϕ_{12} = 0.6 mWb. If N_2 = 1,000 turns and L_2 = 400 mH, find N_1, L_1, M, and ϕ_{22e}. What is the total energy stored in the magnetic circuit? What is the dot convention with respect to i_1 and i_2 implied by the signs of the fluxes?

Solution P6.1.2

$M_{21} = \dfrac{N_2\phi_{21}}{i_1} = \dfrac{N_1\phi_{12}}{i_2} = M_{12}$. It follows that $M = \dfrac{N_2\phi_{21}}{i_1} = \dfrac{1000 \times 0.4}{4} = 100$ mH and

$N_1 = N_2 \dfrac{i_2}{i_1}\dfrac{\phi_{21}}{\phi_{12}} = 1000 \dfrac{3}{4}\dfrac{0.4}{0.6} = 500$ turns. $L_1 = \dfrac{N_1\left(\phi_{11e} + \phi_{21}\right)}{i_1} = \dfrac{500(0.1 + 0.4)}{4} = 62.5$ mH.

$3 \times 400 = 1000(0.6 + \phi_{22e})$. This gives ϕ_{22e} = 0.6 mWb. Or, $\dfrac{(100)^2}{62.5 \times 400} = \dfrac{0.4}{0.1 + 0.4} \times \dfrac{0.6}{\phi_{22e} + 0.6}$,

which gives the same ϕ_{22e}.

Since ϕ_{21} and ϕ_{12} are both positive, they are in the same direction, and the energy stored is $0.5 \times 62.5 \times (4)^2 + 0.5 \times 400 \times (3)^2 + 100 \times 4 \times 3 \equiv 3.5$ mJ. The same sign of ϕ_{21} and ϕ_{12} implies that i_1 and i_2 both enter the dotted terminals.

P6.1.3 Two coils having N_1 = 800 turns and N_2 = 500 turns are coupled through a high-permeability core. A current i_1 in coil 1 produces ϕ_{11e} = 500 μWb and ϕ_{21} = 400 μWb, whereas a current $2i_1$ in coil 2 produces ϕ_{22e} = 1400 μWb. (a) What ϕ_{12} is produced by $2i_1$ in coil 2? (b) What is the coefficient of coupling? (c) If the permeance of the core is 50 nWb/A-turn, what is the mutual inductance? (d) What is the inductance of each coil?

Solution P6.1.3

(a) Since $M_{12} = M_{21}$: $\dfrac{500 \times 400}{i_1} = \dfrac{800 \times \phi_{12}}{2i_1}$, so $\phi_{12} = 500$ μWb.

(b) From Equation 6.1.20, $k = \sqrt{\dfrac{400}{500+400} \times \dfrac{500}{1400+500}} = \dfrac{2}{3}\sqrt{\dfrac{5}{19}} = 0.342$.

(c) $M = \dfrac{N_2 \times \phi_{21}}{i_1} = \dfrac{N_2 \times N_1 i_1 \times P_c}{i_1} = N_1 N_2 P_c = 800 \times 500 \times 50 \times 10^{-9} = 20$ mH.

(d) $L_1 = \dfrac{800 \times 900}{i_1}$, $L_2 = \dfrac{500 \times 1900}{2i_1}$; hence, $\dfrac{L_1}{L_2} = \dfrac{800 \times 900 \times 2}{500 \times 1900} = 1.516$, and $L_1 L_2 =$

$\dfrac{M^2}{k^2} = \left(\dfrac{20}{0.342}\right)^2$. This gives $L_1 = 72$ mH and $L_2 = 47.50$ mH.

P6.1.5 Determine the frequency at which the current i in Fig. P6.1.5 has the same magnitude when the connections of one coil are reversed. Find for both connections: (a) the peak current, and (b) the peak stored magnetic energy, if $v = 100\cos(1{,}000t)$ V.

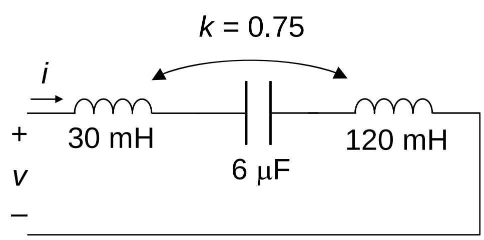


Figure P6.1.5

Solution P6.1.5

$M = 0.75\sqrt{30 \times 120} = 45$ mH.

Since the current magnitude remains the same when one coil is reversed, the impedance reverses sign.

Thus: $j\omega(30 + 120 + 90) - \dfrac{j}{\omega C} = -j\omega(30 + 120 - 90) + \dfrac{j}{\omega C}$, or $300\omega = \dfrac{2}{\omega C}$. $\omega^2 = \dfrac{1}{150 \times 10^{-3} \times 6 \times 10^{-6}}$; $\omega = \dfrac{\sqrt{10}}{3}$ krad/s.

For the maximum inductance connection, at ω = 1000 rad/s

$$Z = j10^3 \times 240 \times 10^{-3} - \frac{j}{10^3 \times 6 \times 10^{-6}} = j220/3\ \Omega;$$

$$\text{Magnitude of peak current} = \frac{100}{220/3} = \frac{15}{11}\ \text{A};$$

$$\text{Peak stored magnetic energy} = 0.5 \times \left(\frac{15}{11}\right)^2 \times 240 \times 10^{-3} = 0.223\ \text{J}.$$

For the minimum inductance connection, at ω = 1000 rad/s

$$Z = j10^3 \times 60 \times 10^{-3} - \frac{j}{10^3 \times 6 \times 10^{-6}} = -j320/3\ \Omega;$$

$$\text{Magnitude of peak current} = \frac{100}{320/3} = \frac{15}{16}\ \text{A};$$

$$\text{Peak stored magnetic energy} = 0.5 \times \left(\frac{15}{16}\right)^2 \times 60 \times 10^{-3} = 0.0264\ \text{J}.$$

P6.2.10 The linear transformer of Figure P6.2.10 has L_1 = 40 mH, L_2 = 60 mH, and M = 20 mH. If $i_1 = 0.5\cos(500t)$ A, determine i_2, v_1, and v_2 both as time functions and as phasors. What is the power dissipated in the circuit?

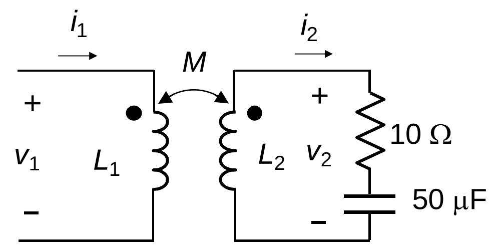
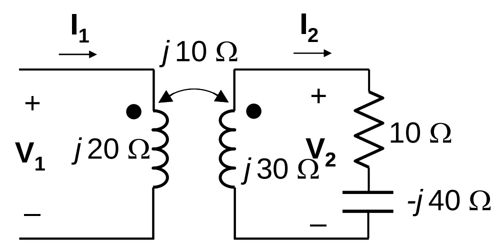


Figure P6.2.10

<u>Solution P6.2.10</u>



$1/j\omega C = 1/(j500\times50\times10^{-6}) = -j40\ \Omega$;

$j\omega M = j500\times20\times10^{-3} = j10\ \Omega$.

The current equations are:

$$j20\mathbf{I_1} - j10\mathbf{I_2} = \mathbf{V_1}$$

and $-j10\mathbf{I_1} + (10 - j10)\mathbf{I_2} = 0$.

The latter equation gives: $\mathbf{I_2} = \dfrac{j}{1-j}\mathbf{I_1} = \dfrac{-1+j}{2}\times 0.5 = 0.25(-1+j) = 0.25\sqrt{2}\angle 135^\circ$ A $\equiv$

$0.25\sqrt{2}\cos(500t + 135^\circ)$ A.

$\mathbf{V_2} = 0.25\sqrt{2}\angle 135^\circ(10 - j40) = 14.58\angle 59^\circ$ V $\equiv 14.58\cos(500t + 59^\circ)$ V.

From the first equation, $\mathbf{V_1} = j20*0.5 - j10\times0.25\times(-1 + j) = 2.5 + j12.5 = 12.75\angle 78.7^\circ$ V $\equiv$ $12.75\cos(500t + 78.7^\circ)$ V.

The power dissipated in the 10 Ω resistor is $(0.25)^2\times10 = 0.625$ W.