Homework 12

P9.2.1 The triangular pulse train of Figure 9.3.2 is applied as the input v_I to the circuit of Fig. P9.2.1. Determine v_O. Show that if $\omega CR << 1$, $v_O \to \frac{dv_I}{dt}$, and v_O becomes the square pulse train of Figure 9.2.8 with appropriate amplitude.

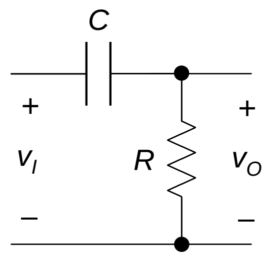

Figure P9.2.1

Solution P9.2.1

From Equation 9.3.13: $v_I = \frac{-8A_m}{\pi^2}\left[\cos\omega_0 t + \frac{1}{9}\cos 3\omega_0 t + \frac{1}{25}\cos 5\omega_0 t + ...\right]$.

In terms of phasors: $\frac{V_O}{V_I} = \frac{R}{R + \frac{1}{j\omega C}} = \frac{j\omega CR}{1 + j\omega CR}$, where $\omega = n\omega_0$.

If $v_I = A\cos\omega t$, then $v_O = \frac{-\omega CR}{\sqrt{1+\omega^2 C^2 R^2}} A\sin(\omega t - \alpha)$, where $\tan\alpha = \omega CR$.

It follows that: $v_O = \frac{8A_m}{\pi^2}\left[\frac{\omega_0 CR}{\sqrt{1+\omega^2 C^2 R^2}}\sin(\omega_0 t - \alpha) + \frac{\omega_0 CR}{3\sqrt{1+9\omega_0^2 C^2 R^2}}\sin(3\omega_0 t - \alpha) + \right.$

$\left.\frac{\omega_0 CR}{5\sqrt{1+25\omega_0^2 C^2 R^2}}\sin(5\omega_0 t - \alpha) + ...\right.$

If $\omega CR << 1$, then $\frac{V_O}{V_I} \cong j\omega CR$, and $v_O(t) = CR\frac{dv_I(t)}{dt}$. The circuit acts as a differentiating circuit.

P9.2.3 In Figure P9.2.3 v_{SRC} is the full-wave rectified waveform of Figure 9.4.1b having T = 1/50 s and an amplitude of 50 V.

Between v_{SRC} and the 4 kΩ load is the LC filter shown whose purpose is to attenuate the AC components, leaving a near DC voltage across the load. Determine the first four nonzero terms in the FSE of v_O and compare with those of v_{SRC}. What is the rms value of the AC component in the first four nonzero terms in the FSE of v_O?

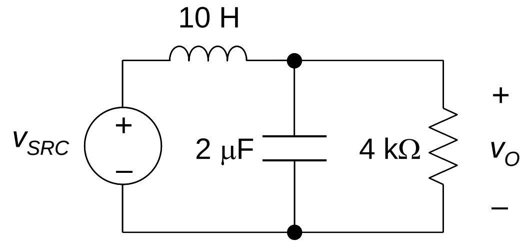

Figure P9.2.3

Solution P9.2.3

$\omega_0 = \dfrac{2\pi}{T} = 100\pi$.

From Equation 9.4.2 $v_{SRC} = \dfrac{100}{\pi} + \dfrac{200}{\pi}\left(\dfrac{1}{3}\cos 200\pi t - \dfrac{1}{15}\cos 400\pi t + \dfrac{1}{35}\cos 600\pi t\right)$ V,

retaining the first four terms.

In terms of phasors: $\dfrac{V_O}{V_{SRC}} = \dfrac{R}{1 + j\omega CR} \times \dfrac{1}{j\omega L + \dfrac{R}{1 + j\omega CR}} = \dfrac{R}{R - \omega^2 LCR + j\omega L}$.

The DC term remains unchanged.

At $\omega = 200\pi$, the first AC term is $\dfrac{200}{3\pi} \times \dfrac{4000}{4000 - (200\pi)^2 \times 10 \times 2 \times 10^{-6} \times 4000 + j2000\pi}$. Its magnitude is $\dfrac{200}{3\pi} \times \dfrac{4}{\sqrt{(4 - 3.2\pi^2)^2 + (2\pi)^2}} = 3$ V and its phase angle is -tan$^{-1}\dfrac{2\pi}{4 - 3.2\pi^2} = 13.1°$.

At $\omega = 400\pi$, the amplitude of the second AC term is $\dfrac{200}{15\pi} \times \dfrac{4}{\sqrt{(4 - 12.8\pi^2)^2 + (4\pi)^2}} = 0.138$ V

and its phase angle is $\tan^{-1}\dfrac{4\pi}{4 - 12.8\pi^2}$ = -5.89°.

At $\omega = 600\pi$, the amplitude of the third AC term is $\dfrac{200}{35\pi} \times \dfrac{4}{\sqrt{(4 - 28.8\pi^2)^2 - (6\pi)^2}} = 0.026$ V and its phase angle is - $\tan^{-1}\dfrac{6\pi}{4 - 28.8\pi^2} = 3.85°$.

The first four terms of the FSE are: $v_O(t) = 31.83 + 3.00\cos(200\pi t + 13.1°) + 0.138\cos(400\pi t - 5.89°) + 0.026\cos(600\pi t + 3.85°)$ V.

Relative rates of attenuation are 1 for the DC term $\frac{3.00}{21.22} = 0.14$ for the fundamental, $\frac{0.138}{4.24} = 0.033$ for the second harmonic, and $\frac{0.026}{1.82} = 0.014$ for the third harmonic.

The rms of AC components of output is $\sqrt{\left(\frac{3}{\sqrt{2}}\right)^2 + \left(\frac{0.138}{\sqrt{2}}\right)^2 + \left(\frac{0.026}{\sqrt{2}}\right)^2} = 2.12$ V.

P9.3.2 Determine the rms value of *i* in Figure P9.3.2.

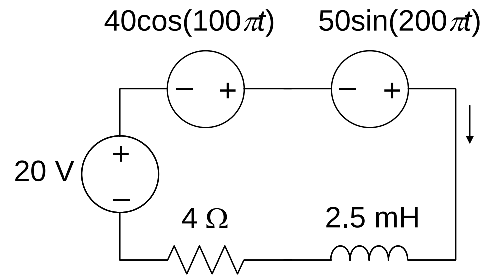

Figure P9.3.2

Solution P9.3.2

Applying superposition, $i_{DC} = \frac{20}{4} = 5$ A;

at $\omega = 100\pi$, $\omega L = 100\pi \times 2.5 \times 10^{-3} = 0.785\ \Omega$, and $i_1 = \frac{40}{\sqrt{2}} \times \frac{1}{\sqrt{(4)^2 + (0.785)^2}}$ = 6.94 A rms;

at $\omega = 200\pi$, $\omega L = 1.57\ \Omega$, and $i_3 = \frac{50}{\sqrt{2}} \times \frac{1}{\sqrt{(4)^2 + (1.57)^2}}$ = 8.23 A rms;

rms value of $i = \sqrt{(5)^2 + (6.94)^2 + (8.23)^2} = 11.87$ A.

P9.3.6 Determine the rms value of the voltage waveform shown in Figure P9.3.6.

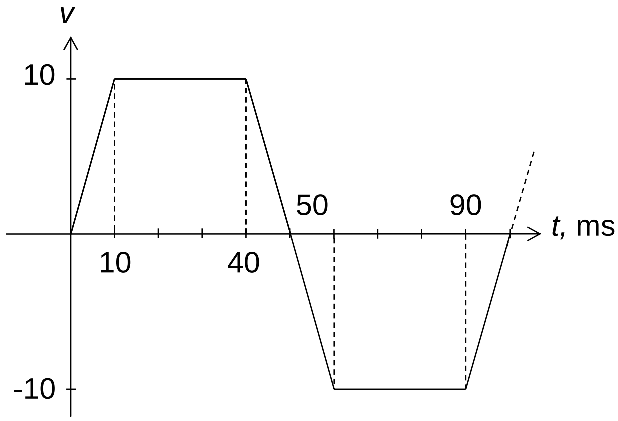

Figure P9.3.6

Solution P9.3.6

The equation of the initial part is *t*.

The area under the square of this function is $\int_0^{10} t^2 dt = \left[\frac{t^3}{3}\right]_0^{10} = \frac{1000}{3}$.

The area under the square of the function from t = 10 s to t = 25 s is 100×15 = 1500; the total area under the quarter period is 5,500/3.

The mean square is 5,500/(25×3) = 220/3.

The rms value is $2\sqrt{55/3}$ V.

P9.3.8 A periodic signal consists of a fundamental of amplitude 20, frequency 1k rad/s and 3rd, 5th, and 7th harmonics of relative amplitudes, with respect to the fundamental of 0.2, 0.05, and 0.01, respectively. The phase angle of the fundamental and 5th harmonics is +90°, whereas the phase angle of the 3rd and 7th harmonics is –90°. (a) Determine the rms value of the signal. (b) Express the signal as a FSE. (c) Specify whether or not the signal is even or odd and whether or not it has half-wave symmetry. Repeat the above for the same signal but with the phase angles of the components negated.

Solution P9.3.8

(a) The amplitudes of the 3^{rd},5^{th}, and 7^{th} harmonics are: 0.2×20 = 4, 0.05×20 = 1, and 0.01×20 = 0.2, respectively. The rms value is $\frac{20}{\sqrt{2}}\sqrt{1+(0.2)^2+(0.05)^2+(0.01)^2}$ = 14.44.

(b) $f(t) = 20\cos(10^3t + 90°) + 4\cos(3\times10^3t - 90°) + \cos(5\times10^3t + 90°) + 0.2\cos(7\times10^3t - 90°) = -20\sin10^3t + 4\sin3\times10^3t - \sin5\times10^3t + 0.2\sin7\times10^3t.$

(c) Since it is a sinusoidal, the function is odd. If we substitute $(t + T/2)$ for t, the function becomes: $-20\sin(10^3t + \pi) + 4\sin(3\times10^3t + 3\pi) - \sin(5\times10^3t + 5\pi) + 0.2(\sin7\times10^3t + 7\pi) = 20\sin10^3t - 4\sin3\times10^3t + \sin5\times10^3t - 0.2\sin7\times10^3t.$ Since the function becomes negated, it is half-wave symmetric.

If the phase angles are negated, $f_1(t) = 20\cos(10^3t - 90°) + 4\cos(3\times10^3t + 90°) + \cos(5\times10^3t - 90°) + 0.2\cos(7\times10^3t + 90°) = 20\sin10^3t - 4\sin3\times10^3t + \sin5\times10^3t - 0.2\sin7\times10^3t.$

Since it is sinusoidal, the function is odd.

If we substitute $(t + T/2)$ for t, the function becomes: $20\sin(10^3t + \pi) - 4\sin(3\times10^3t + 3\pi) + \sin(5\times10^3t + 5\pi) - 0.2(\sin7\times10^3t + 7\pi) = -20\sin10^3t + 4\sin3\times10^3t - \sin5\times10^3t + 0.2\sin7\times10^3t.$

Since the function becomes negated, it is half-wave symmetric.

P9.3.9 The voltage at two given terminals of a circuit is: $v = 4\sin t + 5\cos 2t + 10\sin 4t$ V. The current input at these terminals, in the direction of the voltage drop, is: $i = 6\sin t + 8\cos 5t + 12\sin 8t$ A. Determine: (a) the rms value of v; (b) the rms value of i; (c) the average power delivered to the network at the given terminals.

Solution P9.3.9

(a) The rms value of the voltage is $\sqrt{\left(\frac{4}{\sqrt{2}}\right)^2+\left(\frac{5}{\sqrt{2}}\right)^2+\left(\frac{10}{\sqrt{2}}\right)^2}=8.40\,\text{V}$.

(b) The rms value of the current is $\sqrt{\left(\frac{6}{\sqrt{2}}\right)^2+\left(\frac{8}{\sqrt{2}}\right)^2+\left(\frac{12}{\sqrt{2}}\right)^2}=11.04\ \text{A}$.

(c) The power dissipated is due to the fundamental only and equals $(4\times6)/2 = 12$ W.