

## Homework 5

**P4.1.13** Determine  $V_O$  in Figure P3.1.11 using TEC.

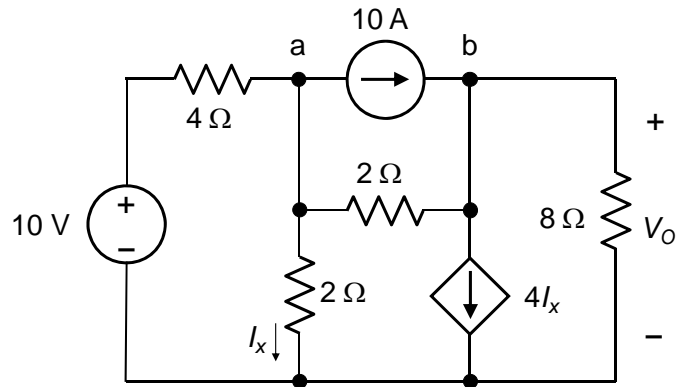


Figure P3.1.11

### Solution P4.1.13

With the  $8\ \Omega$  removed, and the  $10\text{ V}$  source acting alone as an independent source, the current in the  $4\ \Omega$  resistor is  $5I_x$ .

From KVL,  $10 = 20I_x + 2I_x = 22I_x$ , so that  $I_x = 5/11\text{ A}$ ;

$$V_{Th1} = 10 - 20I_x - 8I_x = -30/11\text{ V}.$$

With the  $10\text{ A}$  source acting alone, KCL at node c gives a current of  $5I_x$  flowing toward node a through the  $4\ \Omega$  resistor.

From KVL around the  $4\ \Omega$  and  $2\ \Omega$  resistors,  $20I_x + 2I_x = 0$ , which gives  $I_x = 0$ , so that  $V_{Th2} = 20\text{ V}$ .

$$V_{Th} = 20 - 30/11 = 190/11\text{ V}.$$

When a test source is applied, with the independent

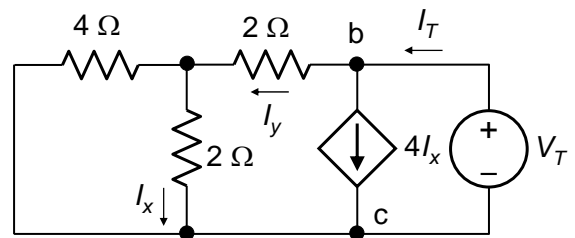
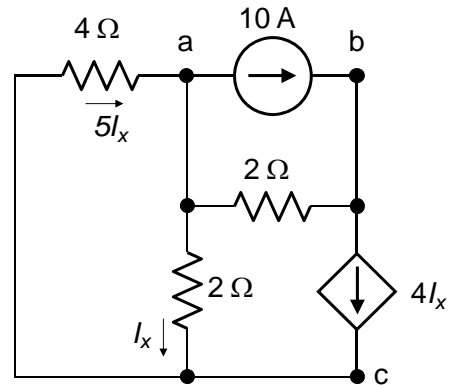
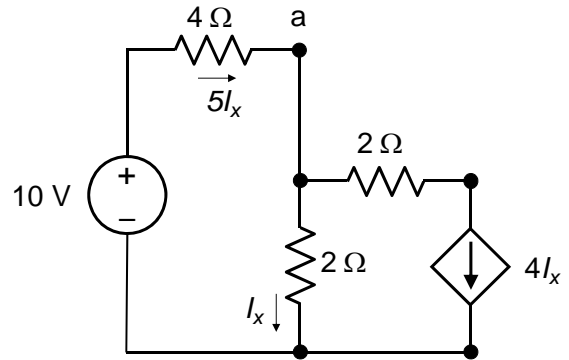
sources set to zero,  $I_x = \frac{2}{3}I_y$ ,

$$V_T = 2I_y + 2I_x = \frac{10}{3}I_y;$$

$$I_T = I_y + 4I_x = \frac{11}{3}I_y;$$

$$R_{Th} = \frac{V_T}{I_T} = \frac{10}{11}\ \Omega.$$

$$\text{It follows that } V_O = \frac{8}{8 + 10/11} \times \frac{190}{11} = \frac{760}{49} = 15.51\text{ V}.$$



**P4.1.14** Determine  $I_O$  in Figure P3.1.13 using NEC.

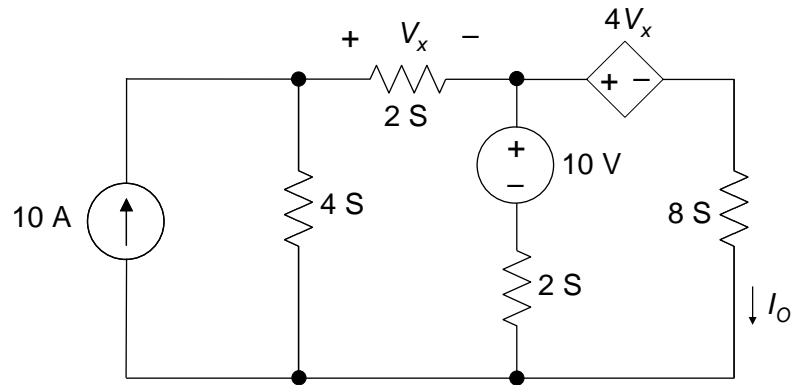


Figure P3.1.13

**Solution P4.1.14**

With the 8 S resistor replaced by a short circuit and the 10 A source acting alone, the voltage across the 4 S resistor is  $5V_x$ .

From KCL,  $10 = 20V_x + 2V_x = 22V_x$ , so that  $V_x = 5/11$  V;

$I_{N1} = 10 - 20V_x - 8V_x = -30/11$  A.

With the 10 V source acting alone, the voltage across the 4 S resistor is  $5V_x$ .

From KCL at the junction between the 4 S and 2 S resistors,  $20V_x + 2V_x = 0$ , which gives  $V_x = 0$ , so that  $I_{N2} = 20$  A, and

$I_N = 20 - 30/11 = 190/11$  A.

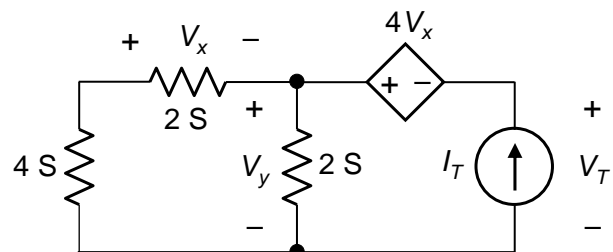
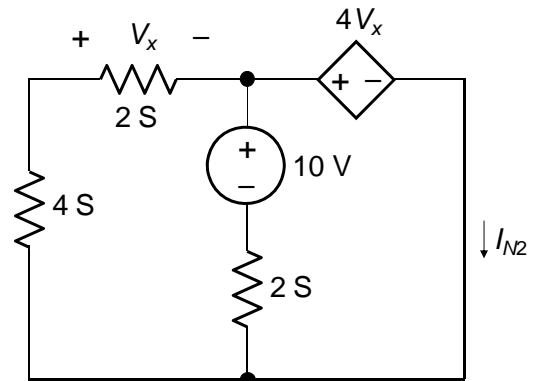
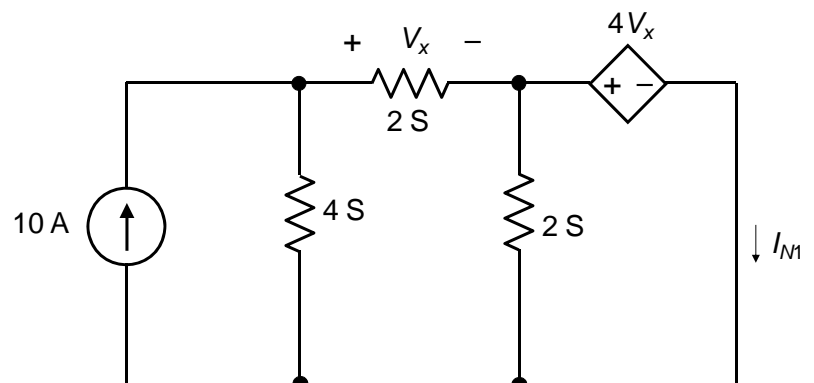
When a test current source is applied, with the independent sources set to zero,  $V_x = -\frac{2}{3}V_y$ ,

$$I_T = 2V_y - 2V_x = \frac{10}{3}V_y;$$

$$V_T = V_y - 4V_x = \frac{11}{3}V_y, \text{ so that } G_N = \frac{I_T}{V_T} = \frac{10}{11} \text{ S.}$$

It follows that

$$I_O = \frac{8}{8 + 10/11} \times \frac{190}{11} = \frac{760}{49} = 15.5 \text{ A.}$$



**P4.2.6** If the current through the dependent source in Figure P3.1.25 is 10 A in the direction of the voltage drop, use the substitution theorem to find  $V_O$ .

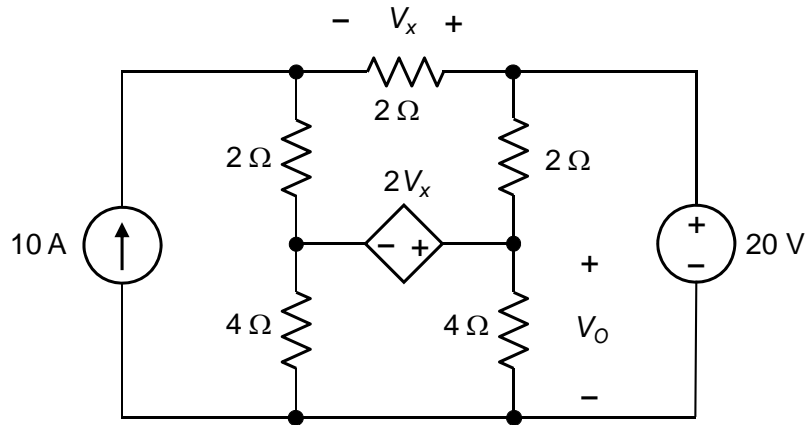


Figure P3.1.25

**Solution P4.2.6**

When the 10 A source on the left acts alone,  $V_O = 0$ .

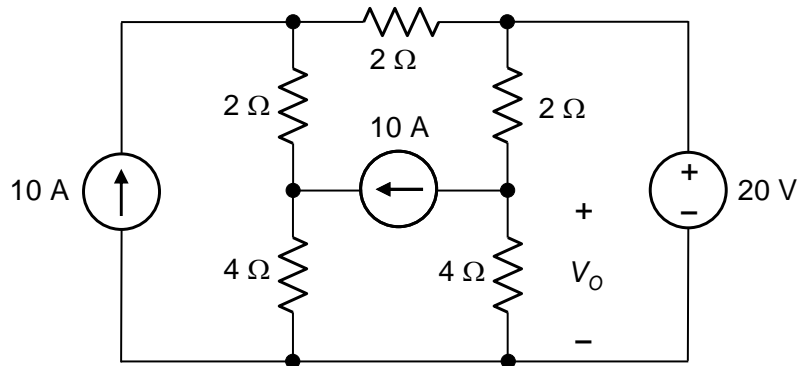
When the 10 A source in the middle acts alone, it is effectively in series with  $2 \parallel 4 \Omega$ , so that  $V_{O1}$

$$= -\frac{40}{3} \text{ V.}$$

When the 20 V source acts

$$\text{alone, } V_{O2} = \frac{4}{6} \times 20 = \frac{40}{3} \text{ V.}$$

It follows that  $V_O = 0$ .



**P4.3.7** Determine the voltage drop from a to b in Figure P4.3.7, assuming that all resistances are  $1\ \Omega$ .

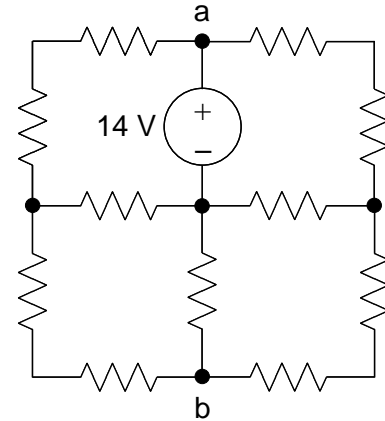


Figure P4.3.7

**Solution P4.3.7**

From symmetry, the corresponding nodes on the same horizontal line are at the same voltage and can be connected together. The circuit becomes folded as shown, with all resistances  $0.5\ \Omega$ , except for the resistor between nodes c and b, which is  $1\ \Omega$ .

The resistance of the lower branches between c and d is  $0.5 \parallel 2 = 2/5\ \Omega$ .

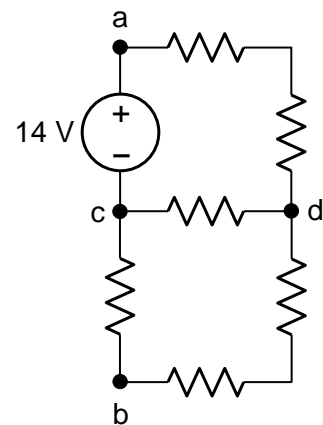
$$\text{Hence } V_{dc} = \frac{14 \times 2/5}{7/5} = 4\ \text{V},$$

$$V_{bc} = \frac{V_{dc}}{2} = 2\ \text{V},$$

$$V_{ab} = 14 - 2 = 12\ \text{V}.$$

Alternatively, the circuit could be split in half along the line ab. The same circuit as previously is obtained, with all resistances  $1\ \Omega$ , except for the resistor between nodes c and b, which is  $2\ \Omega$ .

The same voltages are obtained, since the same resistance ratios apply.



**P4.3.8** Determine the voltage drop from a to b in Figure P4.3.8, assuming that all resistances are  $1\Omega$ .

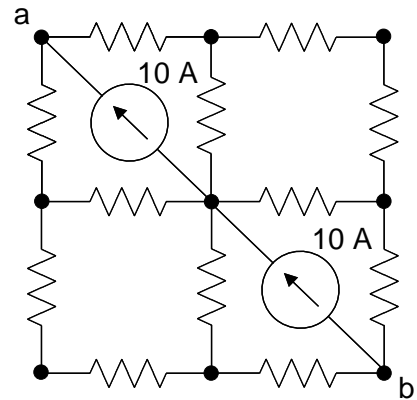


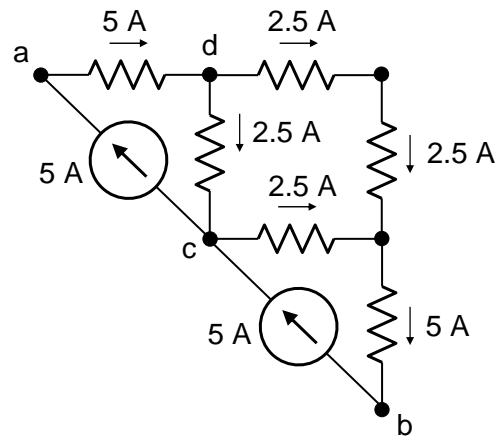
Figure P4.3.8

**Solution P4.3.8**

Since the circuit is symmetrical about the diagonal, a half-circuit may be considered as shown, where all resistances are  $1\Omega$  and the current in branch  $cd$  is  $5\text{ A}$ . At node  $c$ , the net source current is zero, so the two resistances at this node may be disconnected from the sources.

The current entering node  $d$  will then be divided equally at this node.

It follows that  $V_{ab} = 5 + 2.5 + 2.5 + 5 = 15\text{ V}$ .



**P4.3.10** Determine the resistance between terminals a and b in Figure P4.3.10.

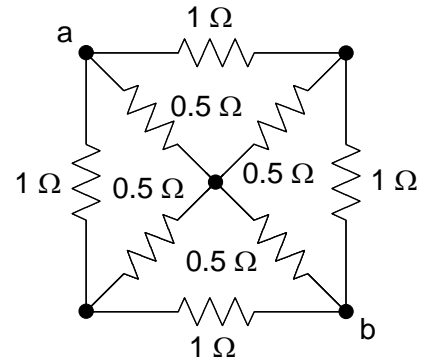
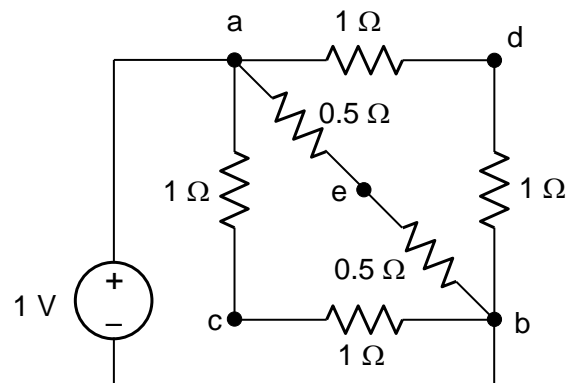


Figure P4.3.10

**Solution P4.3.10**

If a voltage source of say 1 V is applied between ab to determine the resistance, then from symmetry, nodes c, d, and e will be at the same voltage, so that the two 0.5 Ω resistors may be removed. It follows that  $R_{ab} = 2 || 2 || 1 = 1 || 1 \Omega = 0.5 \Omega$ .



**P4.3.11** Determine  
TEC between  
terminals ab in  
Figure P4.3.11  
as well as  
 $I_{SRC1}$ ,  $I_{SRC2}$ ,  
and  $V_{SRC1}$ .

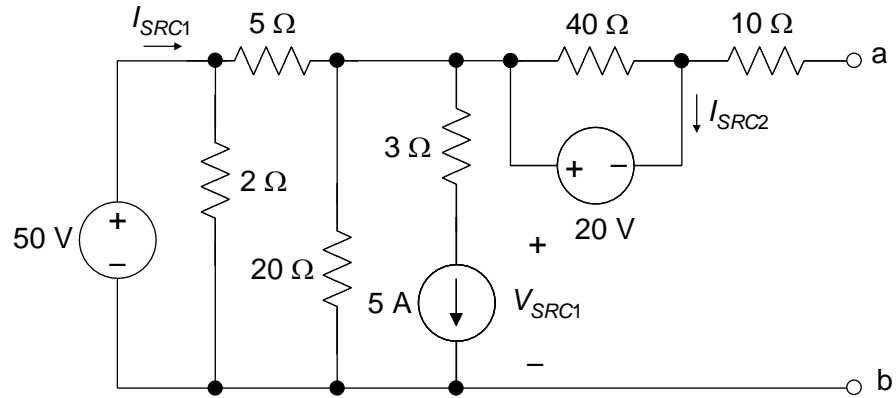
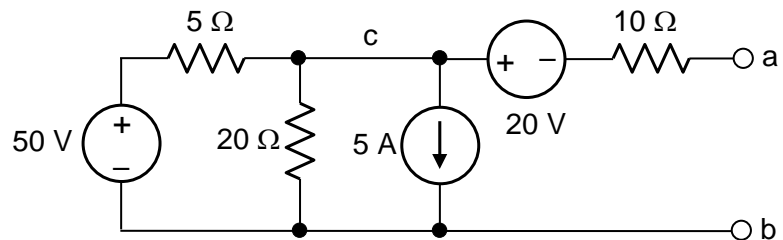


Figure P4.3.11

**Solution P4.3.11**

The resistors in parallel with voltage sources, or in series with the current source are redundant, as far as TEC is concerned.

When removed, the circuit becomes as shown.



The 50 V source is transformed to a 10 A source which combined with the 5 A source results in a 5

A source directed upwards. The voltage of node c with respect to b is  $5(5||20) = 20$  V.

It follows that  $V_{Th} = 0$ .

When the sources are set to zero,  $R_{Th} = 10 + (5||20) = 14 \Omega$ .

With the current at terminal a equal to zero,  $I_{SRC2} = 20/40 = 0.5$  A.

$V_{SRC1} = 20 - 3 \times 5 = 5$  V.

$$I_{SRC1} = \frac{50 - 20}{5} + \frac{50}{2} = 31 \text{ A.}$$